Asymptotes

Consider the graph of the function $f(x) = \dfrac{2x^2}{4 - x^2}$ in Figure 1.32.

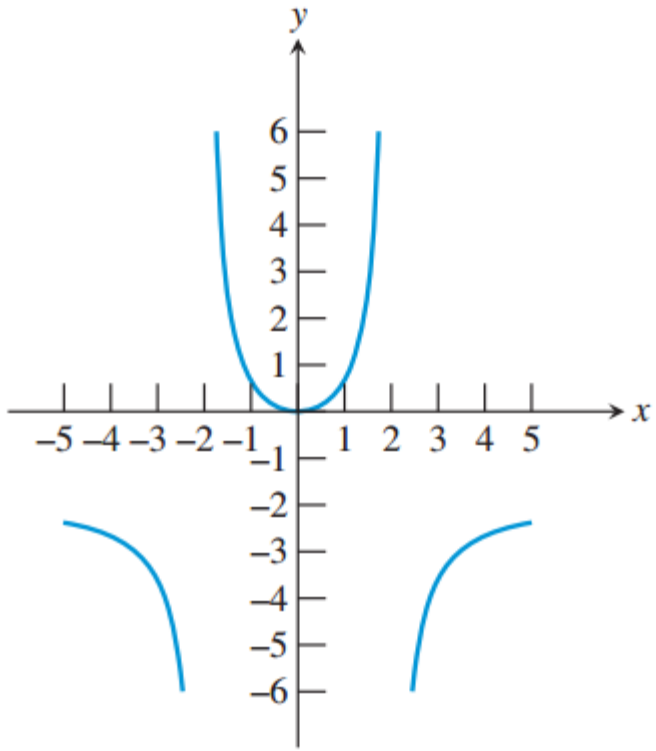


FIGURE 1.32 The graph of $f(x) = 2x^2/(4 - x^2)$ has two vertical asymptotes and one horizontal asymptote.

DEFINITION Horizontal and Vertical Asymptotes

The line $y = b$ is a **horizontal asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of b as x approaches $+\infty$ or $-\infty$.

In limit notation:

$$\lim_{x \to -\infty} f(x) = b \quad \text{or} \quad \lim_{x \to +\infty} f(x) = b.$$

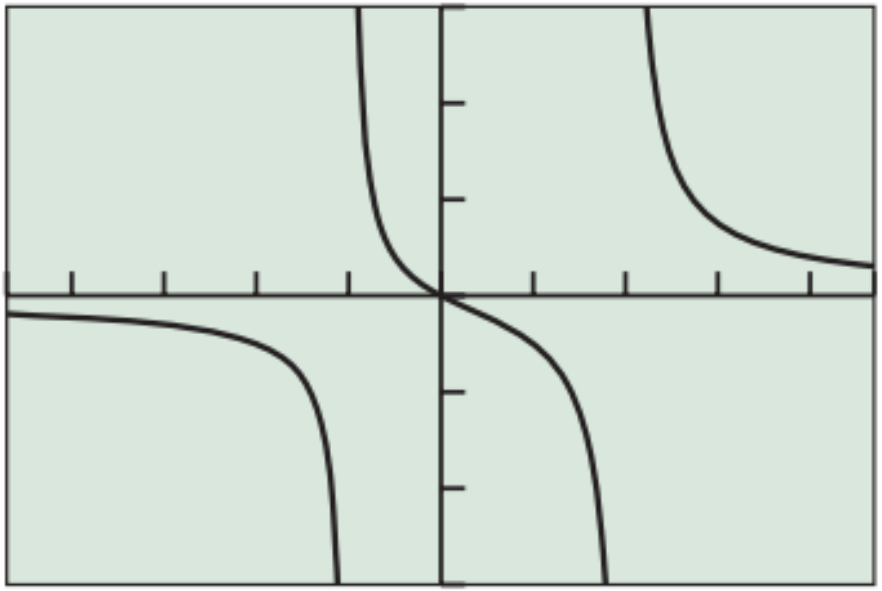
The line $x = a$ is a **vertical asymptote** of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of $+\infty$ or $-\infty$ as x approaches a from either direction.

In limit notation:

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \pm\infty.$$

Identifying the Asymptotes of a Graph

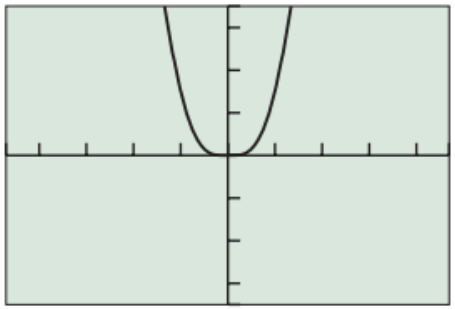
Identify any horizontal or vertical asymptotes of the graph of $y = \dfrac{x}{x^2 - x - 2}$.



[–4.7, 4.7] by [–3, 3]

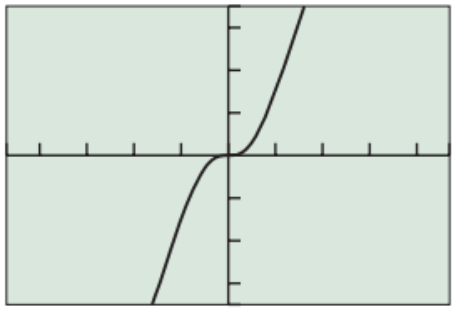
Matching Functions Using End Behavior

(a) $y = \dfrac{3x}{x^2 + 1}$ **(b)** $y = \dfrac{3x^2}{x^2 + 1}$ **(c)** $y = \dfrac{3x^3}{x^2 + 1}$ **(d)** $y = \dfrac{3x^4}{x^2 + 1}$



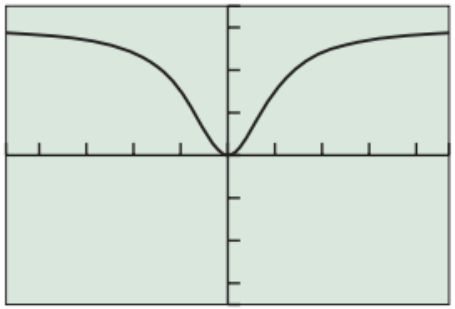
[–4.7, 4.7] by [–3.5, 3.5]

(i)



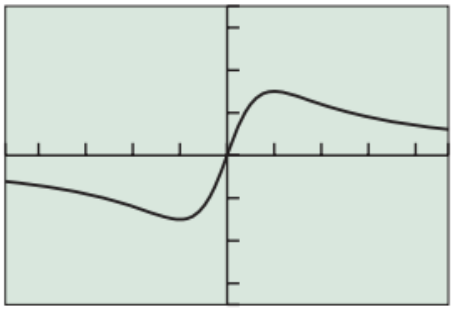
[–4.7, 4.7] by [–3.5, 3.5]

(ii)



[–4.7, 4.7] by [–3.5, 3.5]

(iii)



[–4.7, 4.7] by [–3.5, 3.5]

(iv)