What you'll learn about

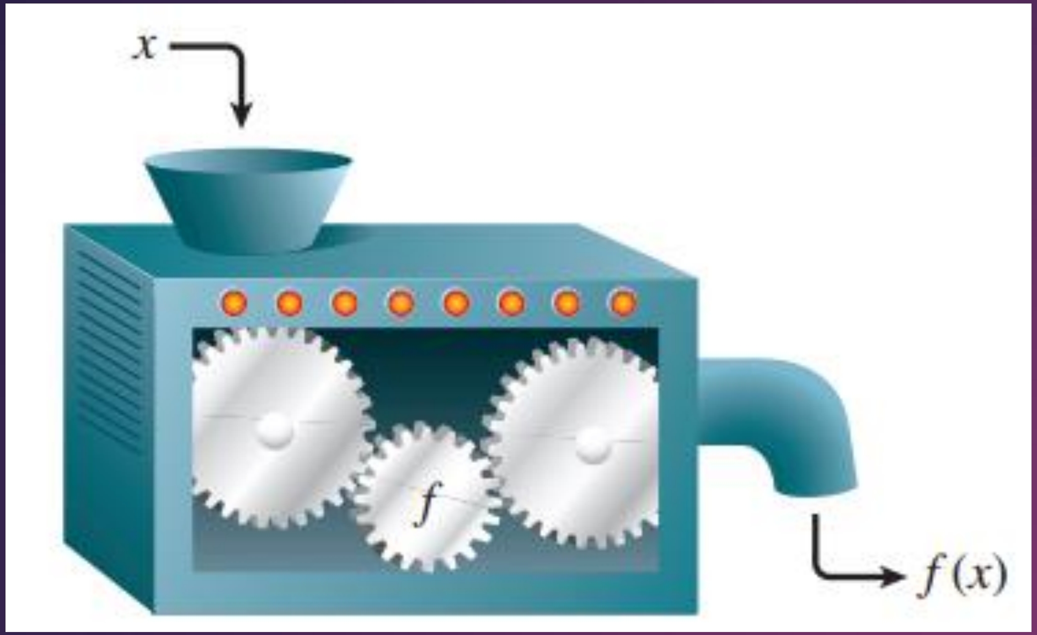
- Function Definition and Notation
- Domain and Range
- Continuity
- Increasing and Decreasing Functions
- Boundedness
- Local and Absolute Extrema
- Symmetry
- Asymptotes
- End Behavior

. . . and why

Functions and graphs form the basis for understanding the mathematics and applications you will see both in your work place and in coursework in college.

DEFINITION Function, Domain, and Range

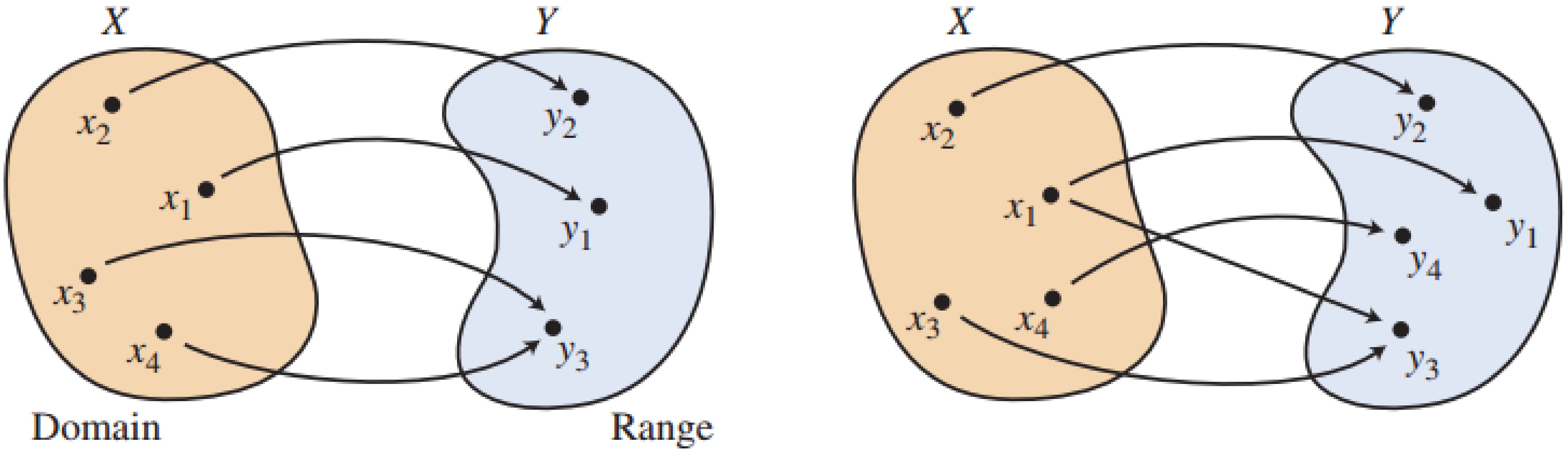
A function from a set D to a set R is a rule that assigns to every element in D a unique element in R. The set D of all input values is the **domain** of the function, and the set R of all output values is the **range** of the function.



There are many ways to look at functions. One of the most intuitively helpful is the "machine" concept (Figure 1.9), in which values of the domain (x) are fed into the machine (the function f) to produce range values (y). To indicate that y comes from the function acting on x, we use Euler's elegant **function notation** $y = f(x)$ (which we read as "***y* equals *f* of *x***" or "**the value of *f* at *x***"). Here x is the **independent variable** and y is the **dependent variable**.

Does the formula $y = x^2$ define y as a function of x?

$V(r) = \frac{4}{3}\pi r^3$ (Note that this is "V of r"—not "$V \cdot r$").



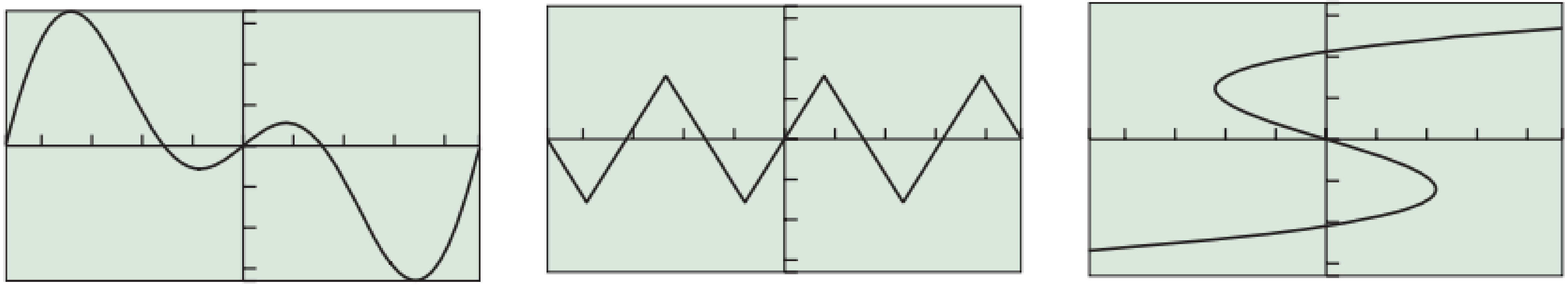
A function

(a)

Not a function

(b)

Seeing a Function Graphically



[–4.7, 4.7] by [–3.3, 3.3]

(a)

[–4.7, 4.7] by [–3.3, 3.3]

(b)

[–4.7, 4.7] by [–3.3, 3.3]

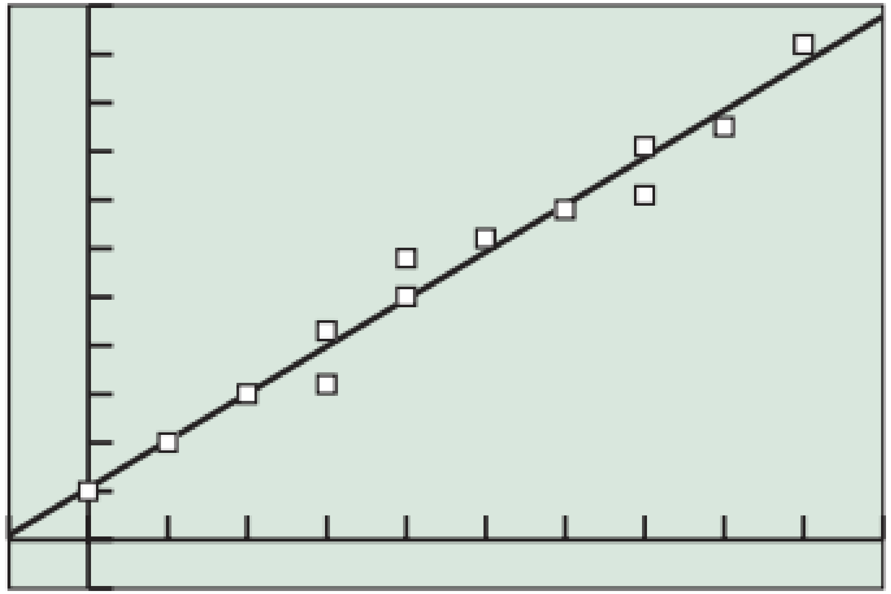
(c)

Vertical Line Test

A graph (set of points (x, y)) in the xy-plane defines y as a function of x if and only if no vertical line intersects the graph in more than one point.

WHAT ABOUT DATA?

When moving from a numerical model to an algebraic model we will often use a function to approximate data pairs that by themselves violate our definition. In Figure 1.12 we can see that several pairs of data points fail the vertical line test, and yet the linear function approximates the data quite well.



[–1, 10] by [–1, 11]

FIGURE 1.12 The data points fail the vertical line test but are nicely approximated by a linear function.

Agreement

Unless we are dealing with a model (like volume) that necessitates a restricted domain, we will assume that the domain of a function defined by an algebraic expression is the same as the domain of the algebraic expression, the **implied domain**. For models, we will use a domain that fits the situation, the **relevant domain**.

$V(r) = \frac{4}{3}\pi r^3$ (Note that this is "V of r"—not "$V \cdot r$").

Finding the Domain of a Function

Find the domain of each of these functions:

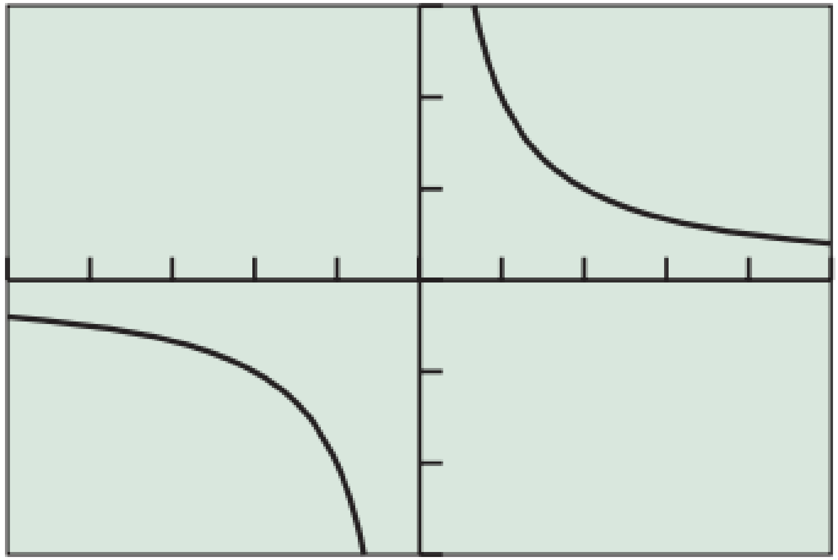
(a) $f(x) = \sqrt{x + 3}$

(b) $g(x) = \dfrac{\sqrt{x}}{x - 5}$

(c) $A(s) = (\sqrt{3}/4)s^2$, where $A(s)$ is the area of an equilateral triangle with sides of length s.

Finding the Range of a Function

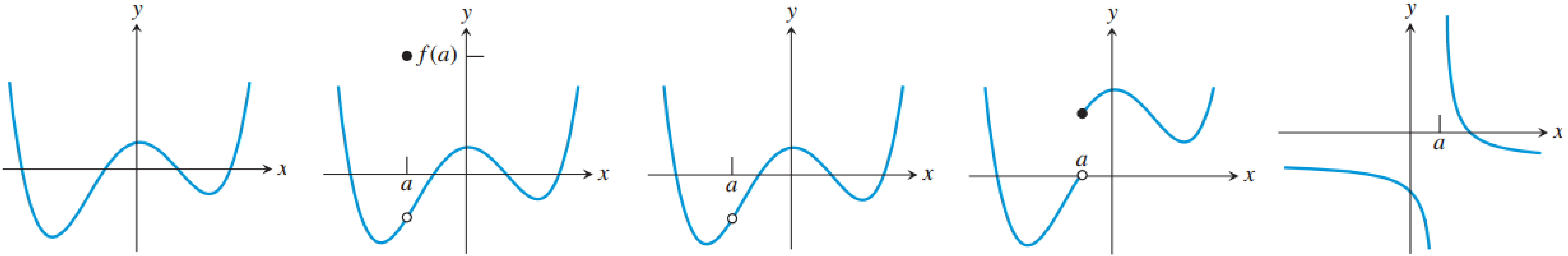
Find the range of the function $f(x) = \dfrac{2}{x}$.



[–5, 5] by [–3, 3]

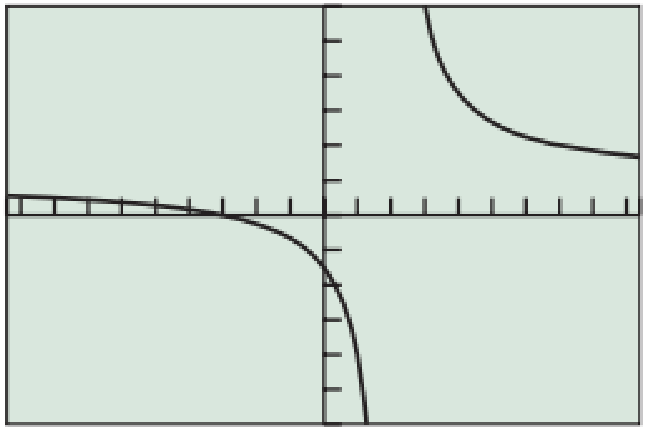
Continuity

One of the most important properties of the majority of functions that model real-world behavior is that they are *continuous*. Graphically speaking, a function is continuous at a point if the graph does not come apart at that point.



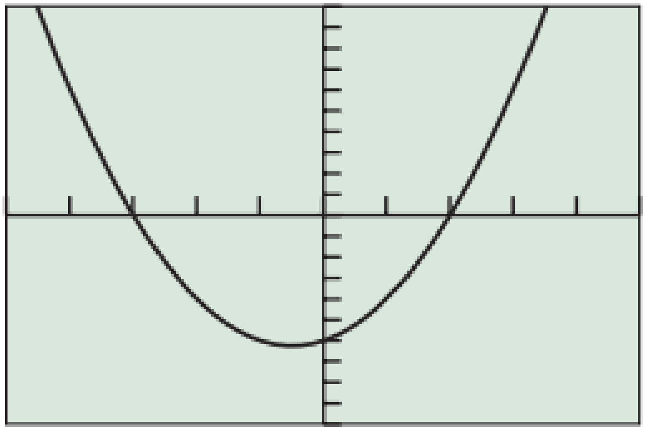
Continuous at all x | Removable discontinuity | Removable discontinuity | Jump discontinuity | Infinite discontinuity

Identifying Points of Discontinuity



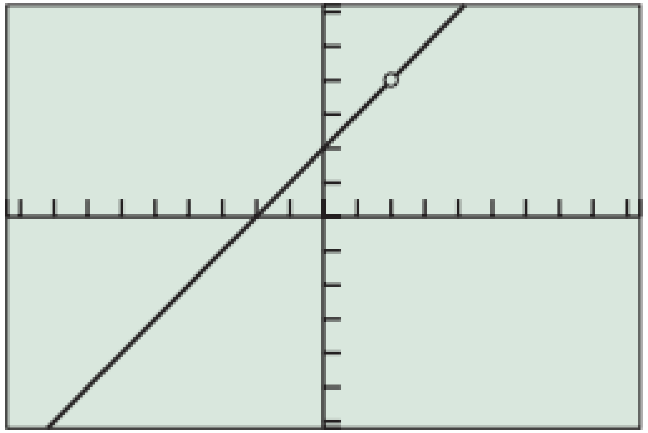
[–9.4, 9.4] by [–6, 6]

FIGURE 1.16 $f(x) = \dfrac{x + 3}{x - 2}$



[–5, 5] by [–10, 10]

FIGURE 1.17 $g(x) = (x + 3)(x - 2)$



[–9.4, 9.4] by [–6.2, 6.2]

FIGURE 1.18 $h(x) = \dfrac{x^2 - 4}{x - 2}$