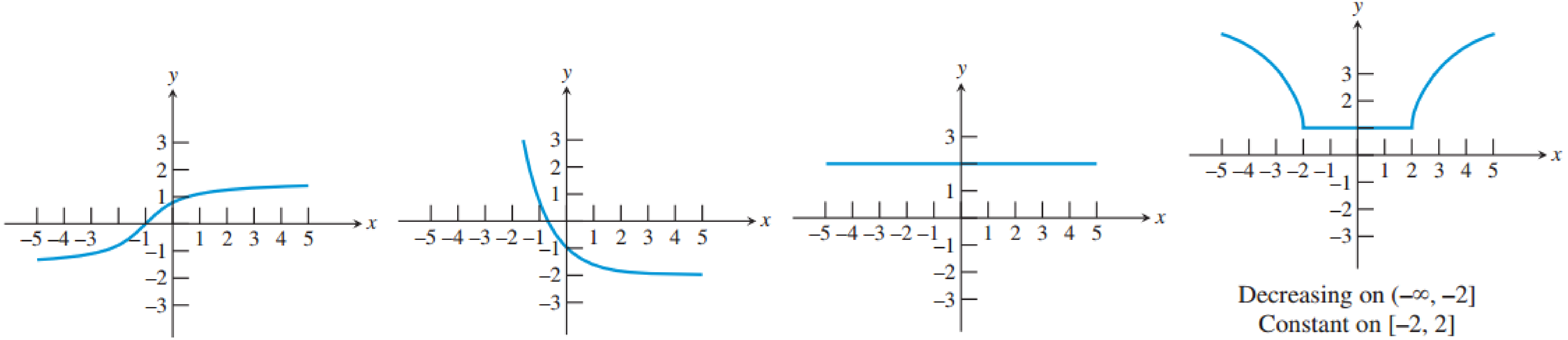
A function f is **continuous at $x = a$** if $\lim_{x \to a} f(x) = f(a)$.

A function f is **discontinuous at $x = a$** if it is not continuous at $x = a$.

Increasing and Decreasing Functions



Increasing

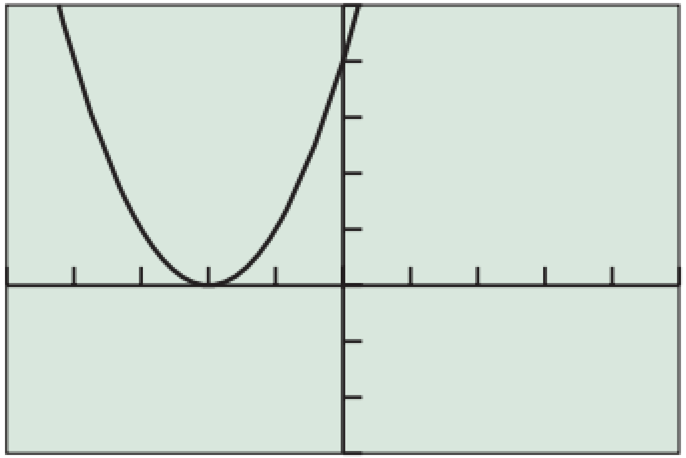
Decreasing

Constant

Decreasing on (–∞, –2]
Constant on [–2, 2]
Increasing on [2, ∞)

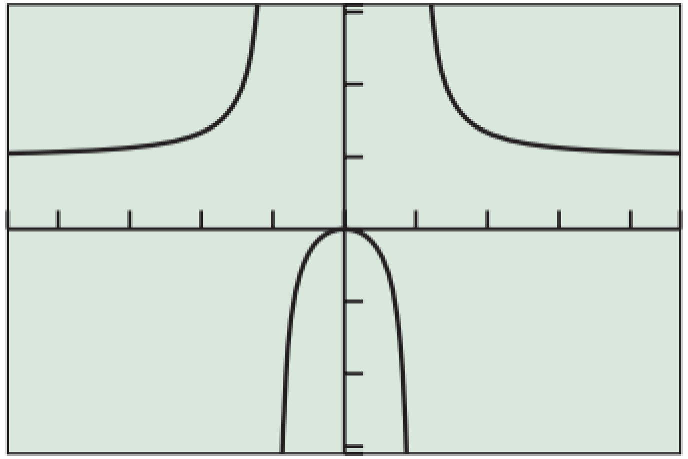
Analyzing a Function for Increasing-Decreasing Behavior

(a) $f(x) = (x + 2)^2$



[–5, 5] by [–3, 5]

(b) $g(x) = \dfrac{x^2}{x^2 - 1}$



[–4.7, 4.7] by [–3.1, 3.1]

Boundedness

The concept of *boundedness* is fairly simple to understand both graphically and algebraically. We will move directly to the algebraic definition after motivating the concept with some typical graphs (Figure 1.22).

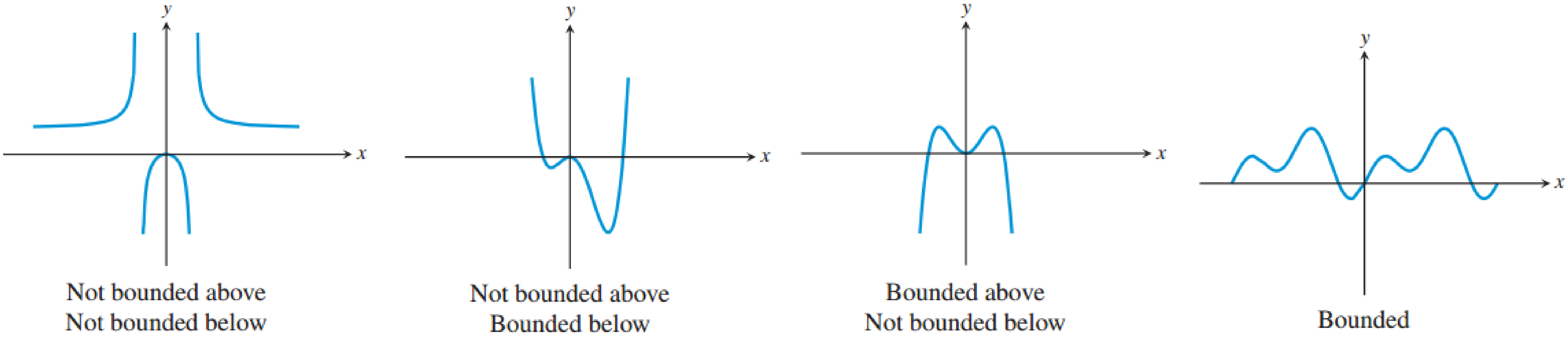


FIGURE 1.22 Some examples of graphs bounded and not bounded above and below.

DEFINITION **Lower Bound, Upper Bound, and Bounded**

A function f is **bounded below** if there is some number b that is less than or equal to every number in the range of f. Any such number b is called a **lower bound** of f.

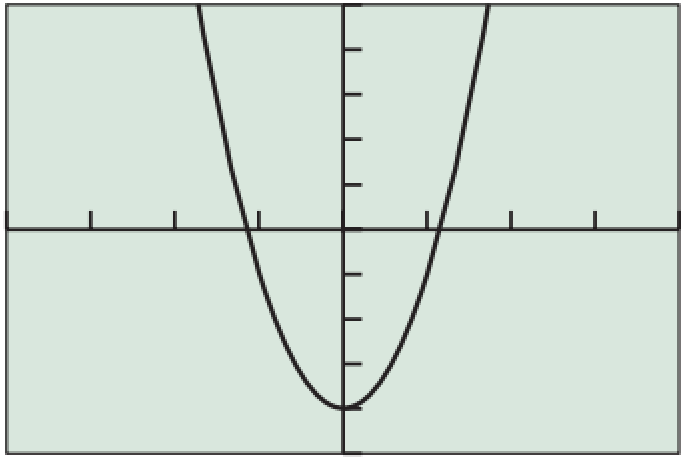
A function f is **bounded above** if there is some number B that is greater than or equal to every number in the range of f. Any such number B is called an **upper bound** of f.

A function f is **bounded** if it is bounded both above and below.

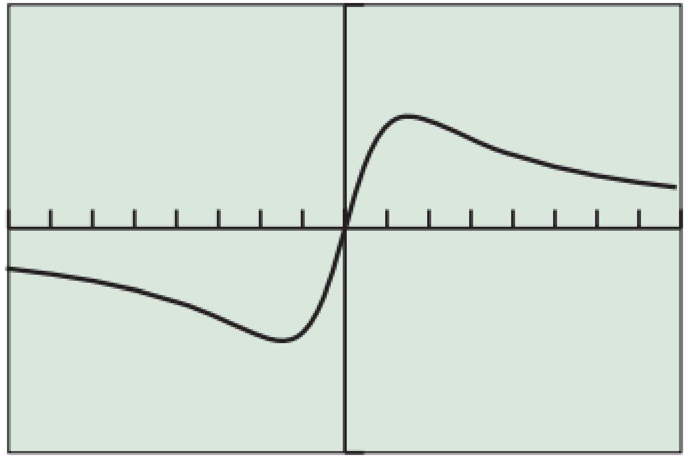
Checking Boundedness

Identify each of these functions as bounded below, bounded above, or bounded.

(a) $w(x) = 3x^2 - 4$ **(b)** $p(x) = \dfrac{x}{1 + x^2}$



[–4, 4] by [–5, 5]
(a)



[–8, 8] by [–1, 1]
(b)

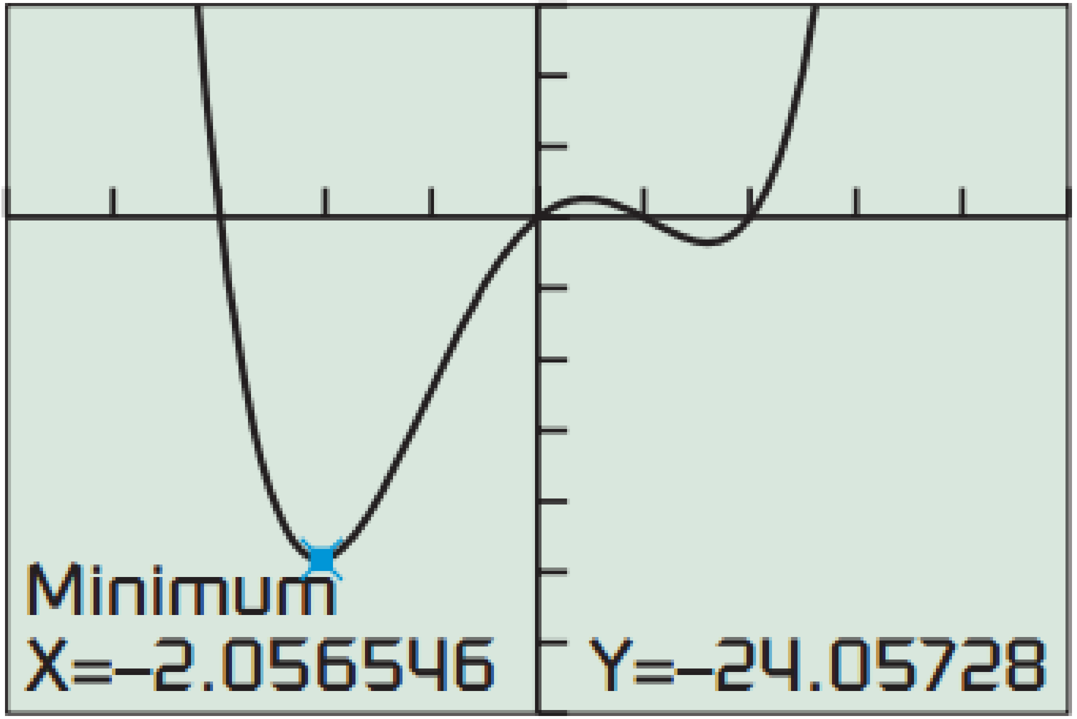
The two graphs are shown in Figure 1.23. It appears that w is bounded below, and p is bounded.

DEFINITION Local and Absolute Extrema

A **local maximum** of a function f is a value $f(c)$ that is greater than or equal to all range values of f on some open interval containing c. If $f(c)$ is greater than or equal to all range values of f, then $f(c)$ is the **maximum** (or **absolute maximum**) value of f.

A **local minimum** of a function f is a value $f(c)$ that is less than or equal to all range values of f on some open interval containing c. If $f(c)$ is less than or equal to all range values of f, then $f(c)$ is the **minimum** (or **absolute minimum**) value of f.

Local extrema are also called **relative extrema**.



[–5, 5] by [–35, 15]

Symmetry

Symmetry with respect to the y-axis
Example: $f(x) = x^2$

Graphically

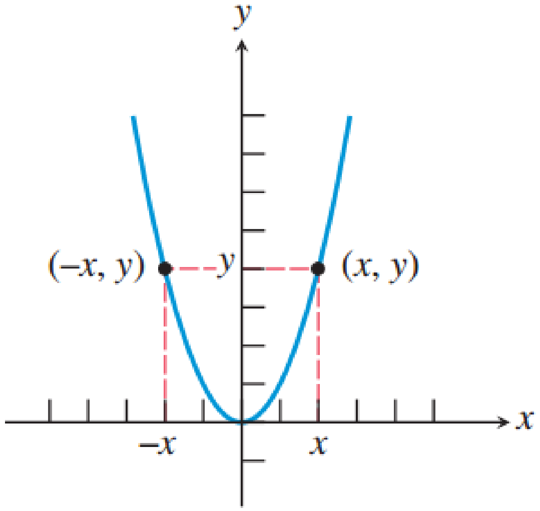


FIGURE 1.26 The graph looks the same to the left of the y-axis as it does to the right of it.

Numerically

x	$f(x)$
−3	9
−2	4
−1	1
1	1
2	4
3	9

Algebraically

For all x in the domain of f,

$$f(-x) = f(x)$$

Functions with this property (for example, x^n, n even) are **even** functions.

Symmetry with respect to the x-axis
Example: $x = y^2$

Graphically

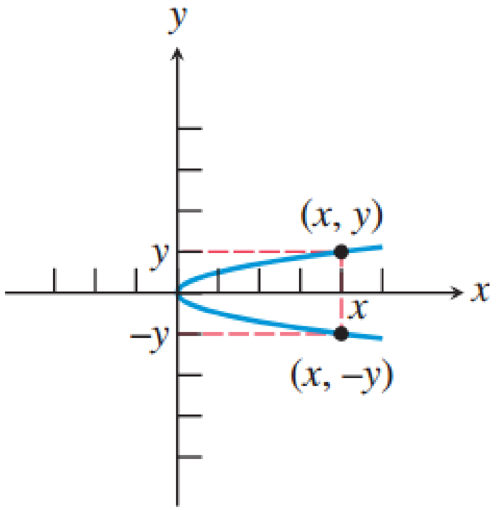


FIGURE 1.27 The graph looks the same above the x-axis as it does below it.

Numerically

x	y
9	−3
4	−2
1	−1
1	1
4	2
9	3

Algebraically

Graphs with this kind of symmetry are not functions (except the zero function), but we can say that $(x, -y)$ is on the graph whenever (x, y) is on the graph.

Symmetry with respect to the origin
Example: $f(x) = x^3$

Graphically

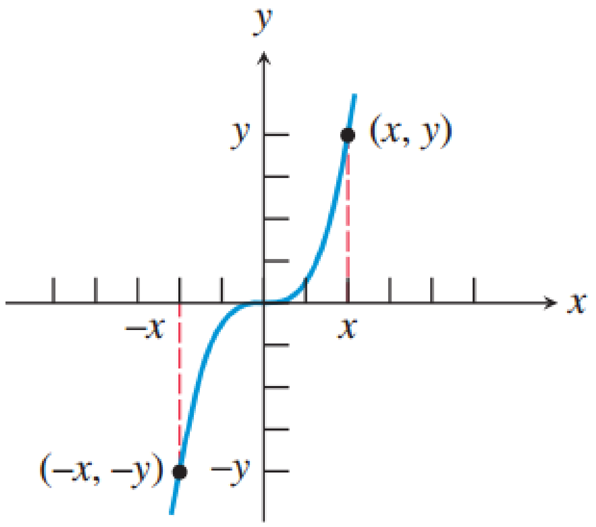


FIGURE 1.28 The graph looks the same upside-down as it does rightside-up.

Numerically

x	y
-3	-27
-2	-8
-1	-1
1	1
2	8
3	27

Algebraically

For all x in the domain of f,

$$f(-x) = -f(x).$$

Functions with this property (for example, x^n, n odd) are **odd** functions.

Checking Functions for Symmetry

Tell whether each of the following functions is odd, even, or neither.

(a) $f(x) = x^2 - 3$ **(b)** $g(x) = x^2 - 2x - 2$ **(c)** $h(x) = \dfrac{x^3}{4 - x^2}$