Building Functions from Functions

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with intersecting domains. Then for all values of x in the intersection, the algebraic combinations of f and g are defined by the following rules:

Sum: $\qquad\qquad (f + g)(x) = f(x) + g(x)$

Difference: $\qquad (f - g)(x) = f(x) - g(x)$

Product: $\qquad\qquad (fg)(x) = f(x)g(x)$

Quotient: $\qquad\qquad \left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, provided $g(x) \neq 0$

In each case, the domain of the new function consists of all numbers that belong to both the domain of f and the domain of g. As noted, the zeros of the denominator are excluded from the domain of the quotient.

Defining New Functions Algebraically

$$\text{Let } f(x) = x^2 \text{ and } g(x) = \sqrt{x + 1}.$$

Find formulas for the functions $f + g, f - g, fg, f/g,$ and gg. Give the domain of each.

$$(f + g)(x) = f(x) + g(x) = x^2 + \sqrt{x + 1}$$

$$(f - g)(x) = f(x) - g(x) = x^2 - \sqrt{x + 1}$$

$$(fg)(x) = f(x)g(x) = x^2\sqrt{x + 1}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{\sqrt{x + 1}}$$
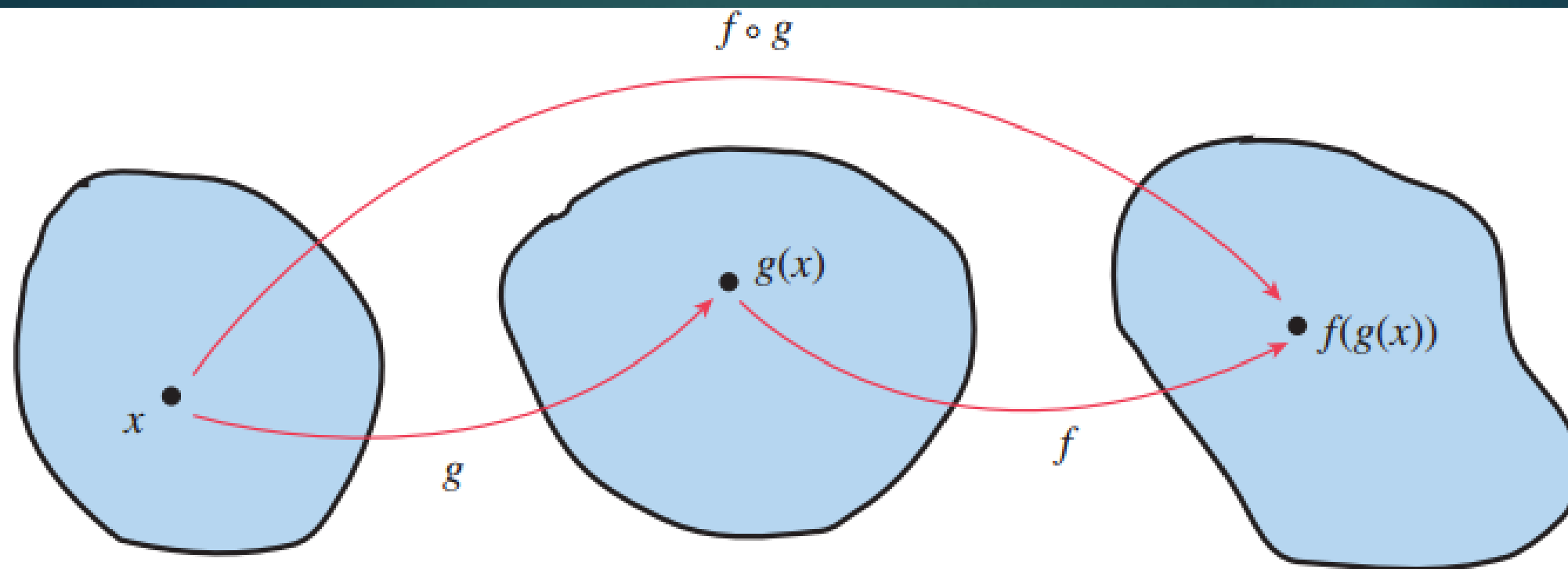
$$(gg)(x) = g(x)g(x) = (\sqrt{x + 1})^2$$

Composition of Functions

Composition of Functions

Let f and g be two functions such that the domain of f intersects the range of g. The **composition f of g**, denoted $f \circ g$, is defined by the rule

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of all x-values in the domain of g that map to $g(x)$-values in the domain of f. (See Figure 1.55.)

FIGURE 1.55 In the composition $f \circ g$, the function g is applied first and then f. This is the reverse of the order in which we read the symbols.
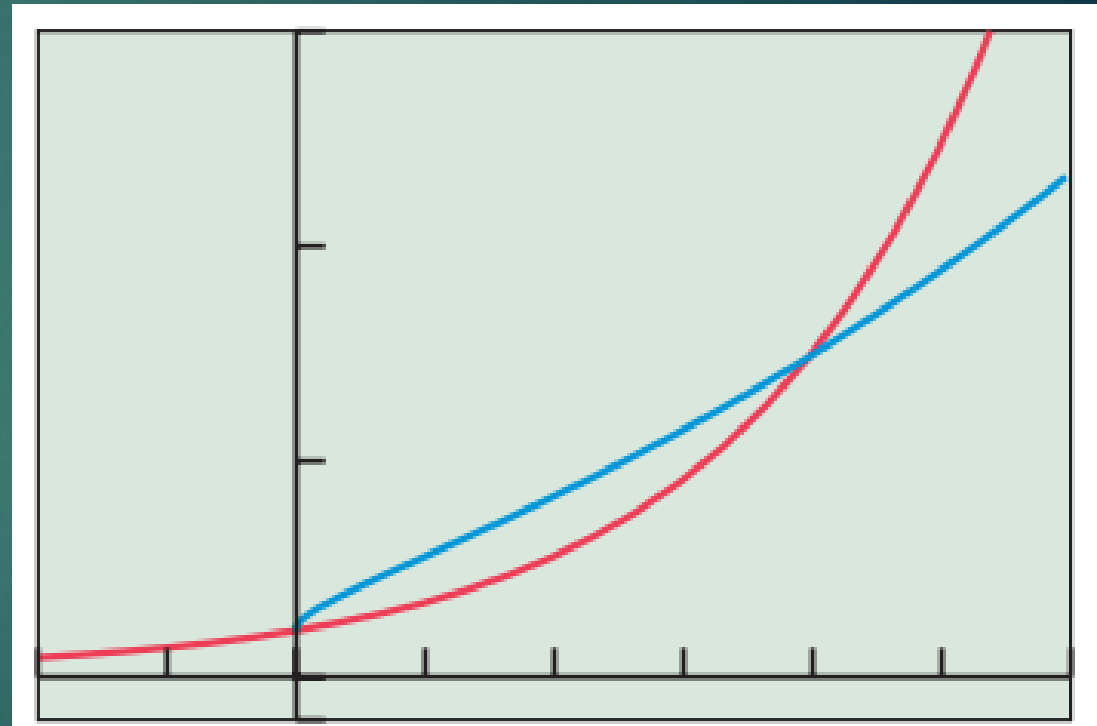
Composing Functions

Let $f(x) = e^x$ and $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ and verify numerically that the functions $f \circ g$ and $g \circ f$ are not the same.

SOLUTION

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = e^{\sqrt{x}}$$

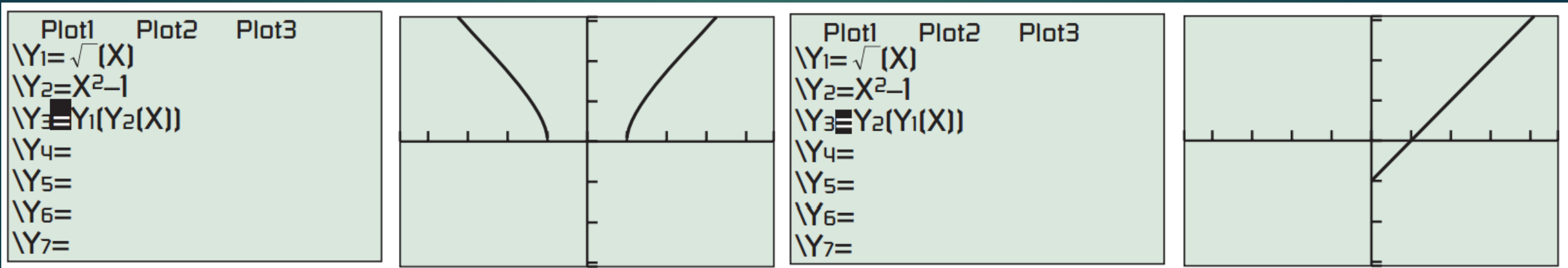$$(g \circ f)(x) = g(f(x)) = g(e^x) = \sqrt{e^x}$$

Finding the Domain of a Composition

Let $f(x) = x^2 - 1$ and let $g(x) = \sqrt{x}$. Find the domains of the composite functions

(a) $g \circ f$

(b) $f \circ g$

Plot1　Plot2　Plot3
\Y1=√(X)
\Y2=X²–1
\Y3=Y1(Y2(X))
\Y4=
\Y5=
\Y6=
\Y7=

(a) $g \circ f$

Plot1　Plot2　Plot3
\Y1=√(X)
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\Y3=Y2(Y1(X))
\Y4=
\Y5=
\Y6=
\Y7=

(b) $f \circ g$

Decomposing Functions

For each function h, find functions f and g such that $h(x) = f(g(x))$.

(a) $h(x) = (x + 1)^2 - 3(x + 1) + 4$

(b) $h(x) = \sqrt{x^3 + 1}$