The Inverse Reflection Principle

The points (a, b) and (b, a) in the coordinate plane are symmetric with respect to the line $y = x$. The points (a, b) and (b, a) are **reflections** of each other across the line $y = x$.

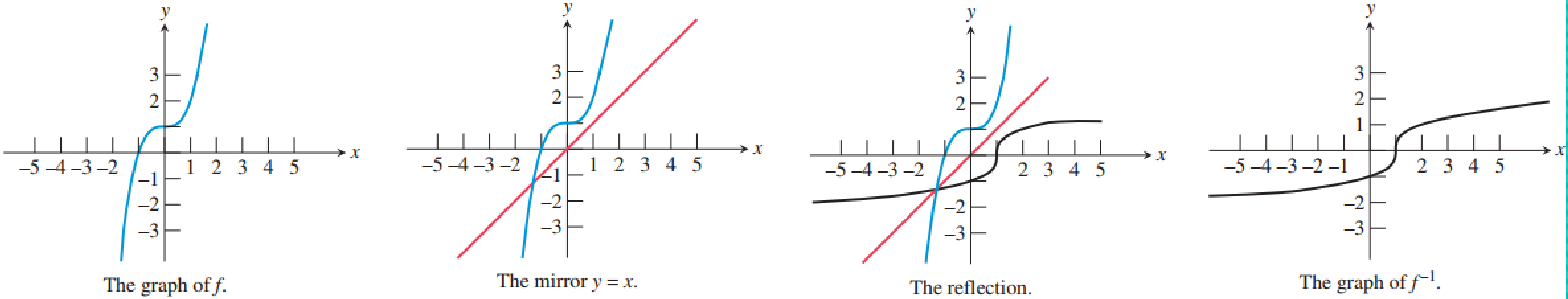


FIGURE 1.69 The Mirror Method. The graph of f is reflected in an imaginary mirror along the line $y = x$ to produce the graph of f^{-1}. (Example 5)

The Inverse Composition Rule

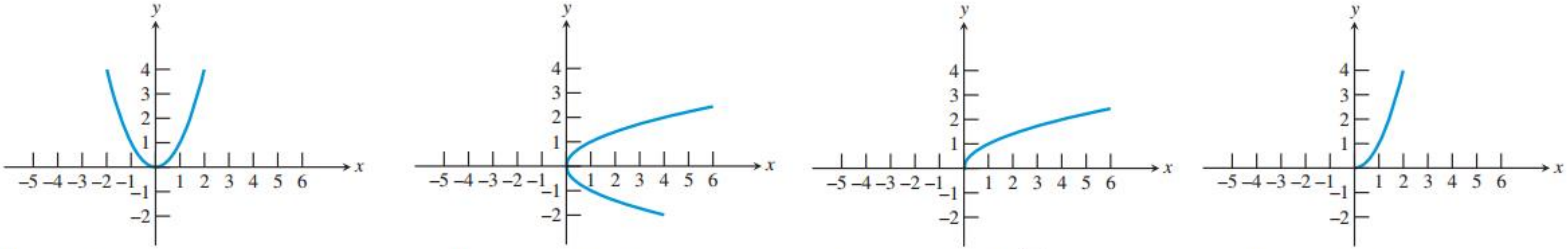
A function f is one-to-one with inverse function g if and only if

$$f(g(x)) = x \text{ for every } x \text{ in the domain of } g \text{, and}$$

$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f.$$

Verifying Inverse Functions

Show algebraically that $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$ are inverse functions.



The graph of $y = x^2$ (not one-to-one).

The inverse relation of $y = x^2$ (not a function).

The graph of $y = \sqrt{x}$ (a function).

The graph of the function whose inverse is $y = \sqrt{x}$.

FIGURE 1.70 The function $y = x^2$ has no inverse function, but $y = \sqrt{x}$ is the inverse function of $y = x^2$ on the restricted domain $[0, \infty)$.

How to Find an Inverse Function Algebraically

Given a formula for a function f, proceed as follows to find a formula for f^{-1}.

1. Determine that there is a function f^{-1} by checking that f is one-to-one. State any restrictions on the domain of f. (Note that it might be necessary to impose some to get a one-to-one version of f.)
2. Switch x and y in the formula $y = f(x)$.
3. Solve for y to get the formula $y = f^{-1}(x)$. State any restrictions on the domain of f^{-1}.

Finding an Inverse Function

Show that $f(x) = \sqrt{x + 3}$ has an inverse function and find a rule for $f^{-1}(x)$. State any restrictions on the domains of f and f^{-1}.