Translations

Let c be a positive real number. Then the following transformations result in translations of the graph of $y = f(x)$:

Horizontal Translations

$y = f(x - c)$	a translation to the right by c units
$y = f(x + c)$	a translation to the left by c units

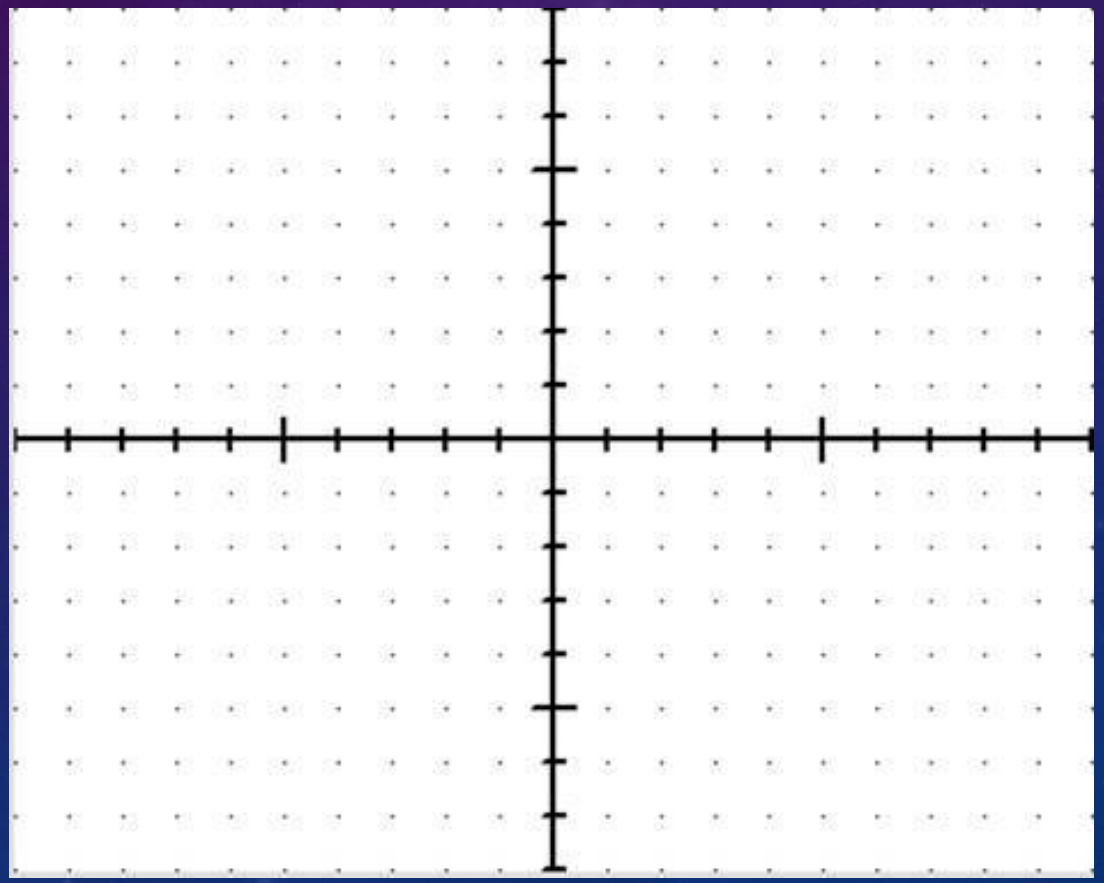
Vertical Translations

$y = f(x) + c$	a translation up by c units
$y = f(x) - c$	a translation down by c units

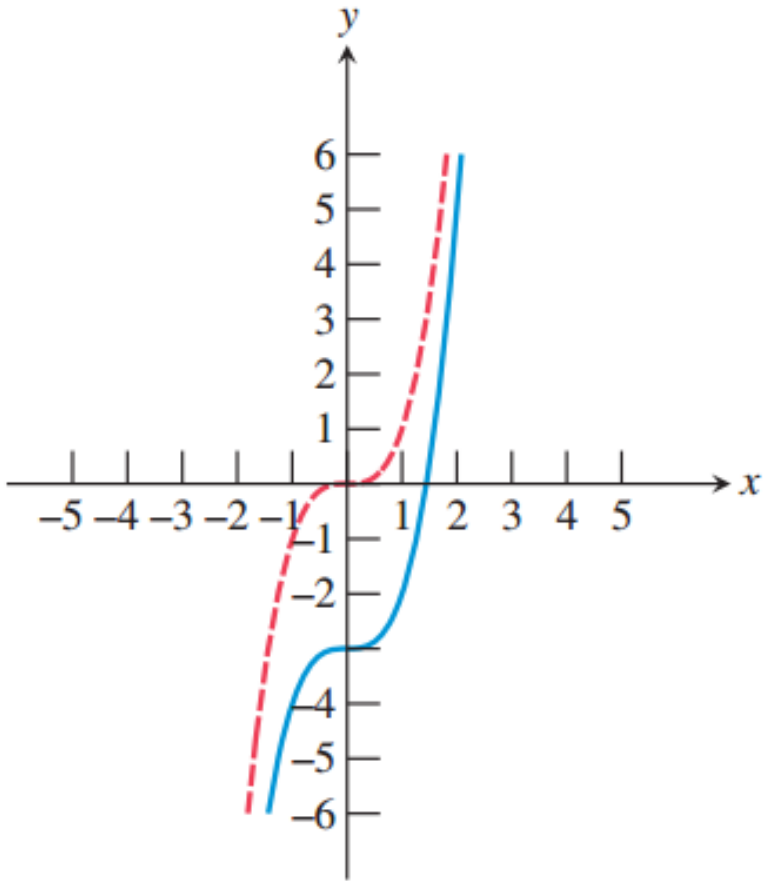
Vertical Translations

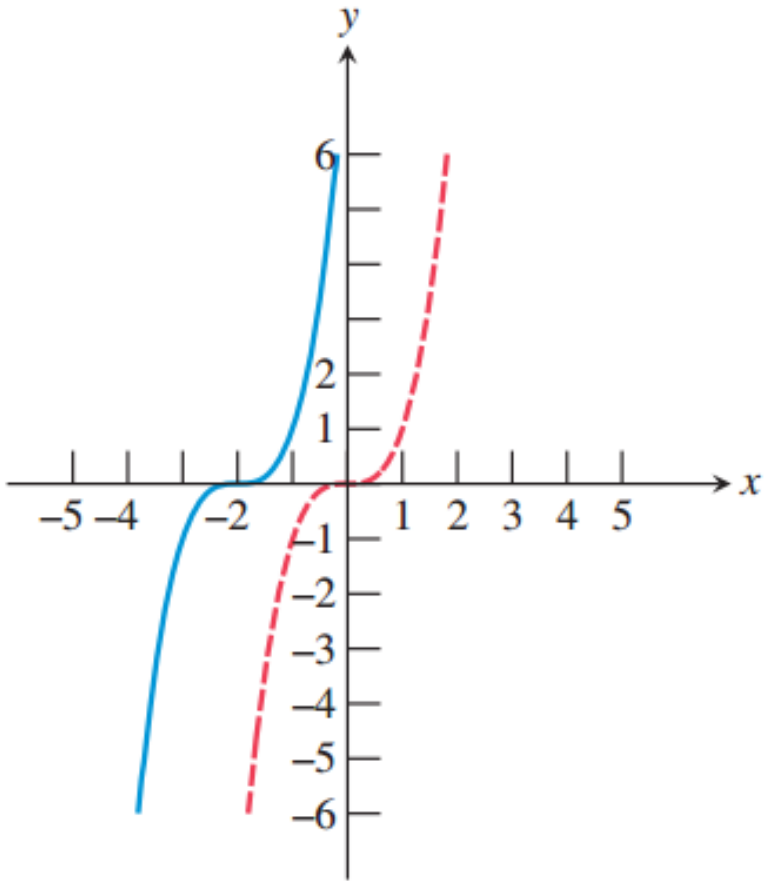
Describe how the graph of $y = |x|$ can be transformed to the graph of the given equation.

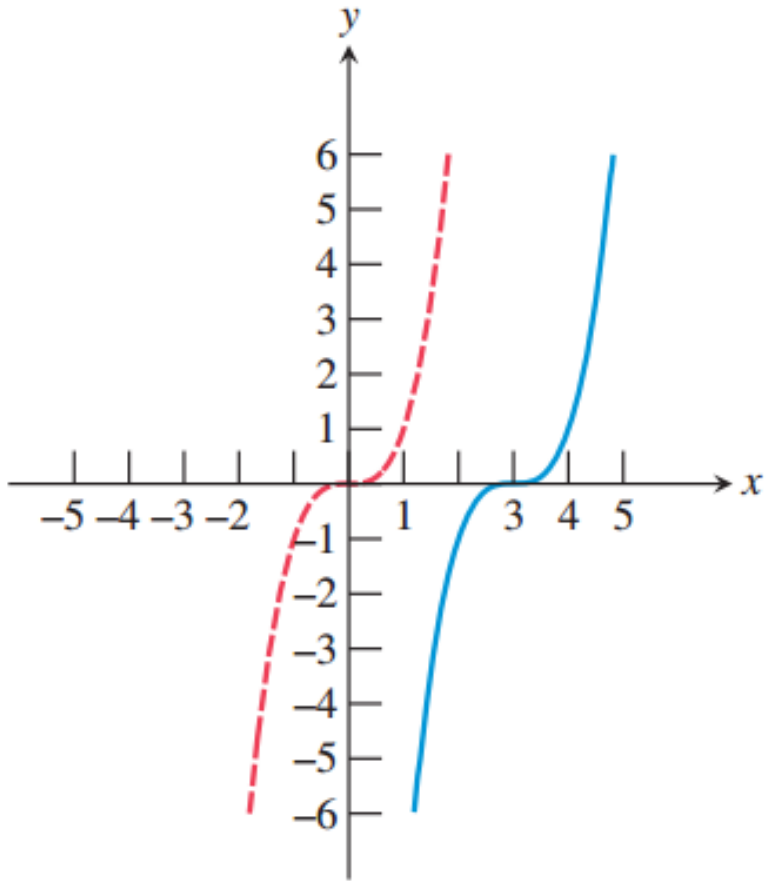
(a) $y = |x| - 4$ **(b)** $y = |x + 2|$



Finding Equations for Translations

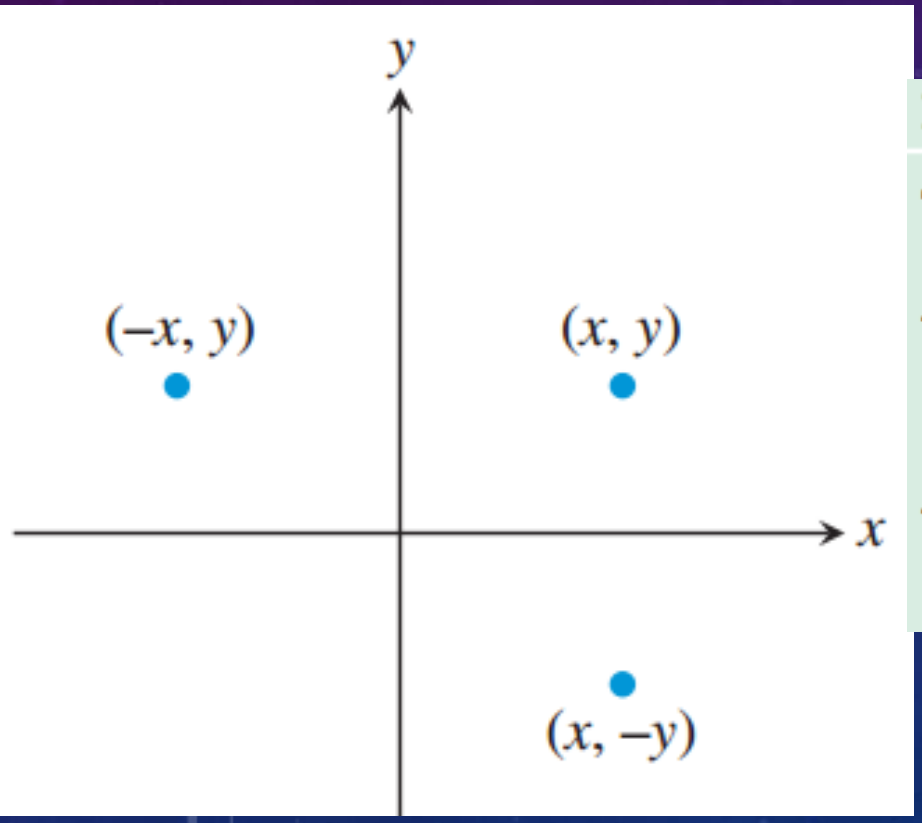






Reflections Across Axes

Points (x, y) and $(x, -y)$ are **reflections of each other across the *x*-axis.** Points (x, y) and $(-x, y)$ are **reflections of each other across the *y*-axis.** (See Figure 1.75.) Two points (or graphs) that are symmetric with respect to a line are **reflections of each other across that line.**



Reflections

The following transformations result in reflections of the graph of $y = f(x)$:

Across the *x*-axis

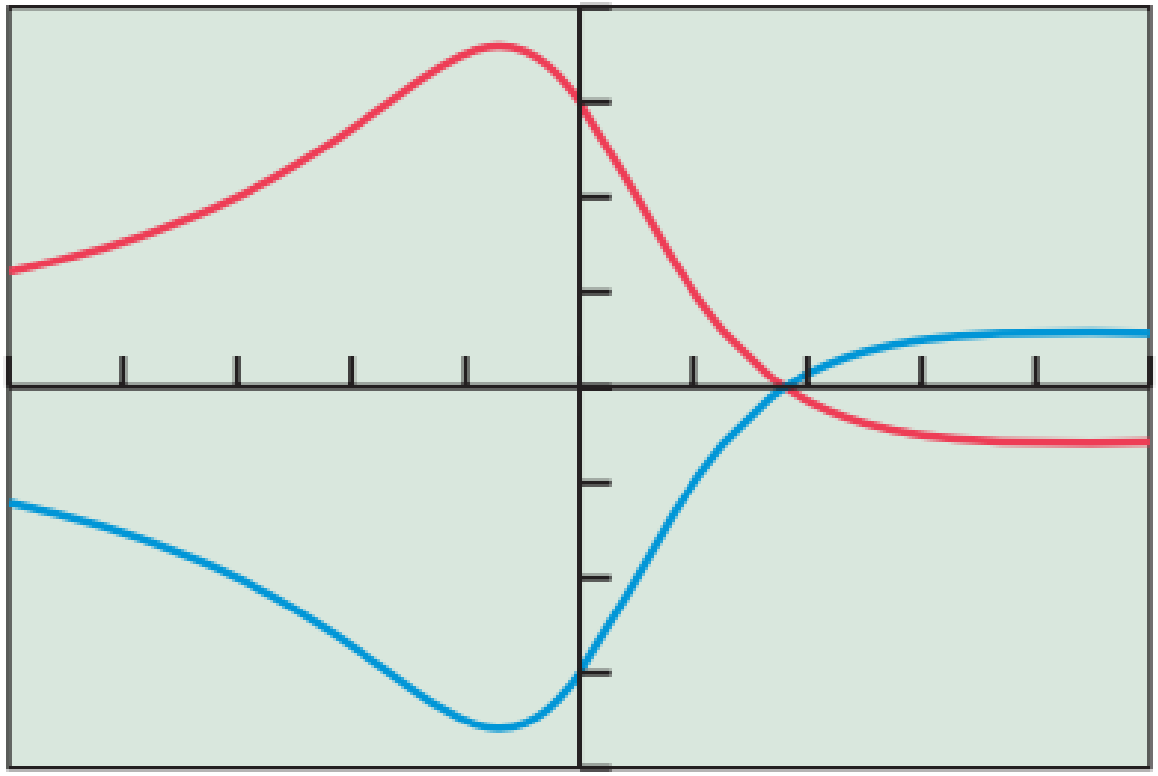
$$y = -f(x)$$

Across the *y*-axis

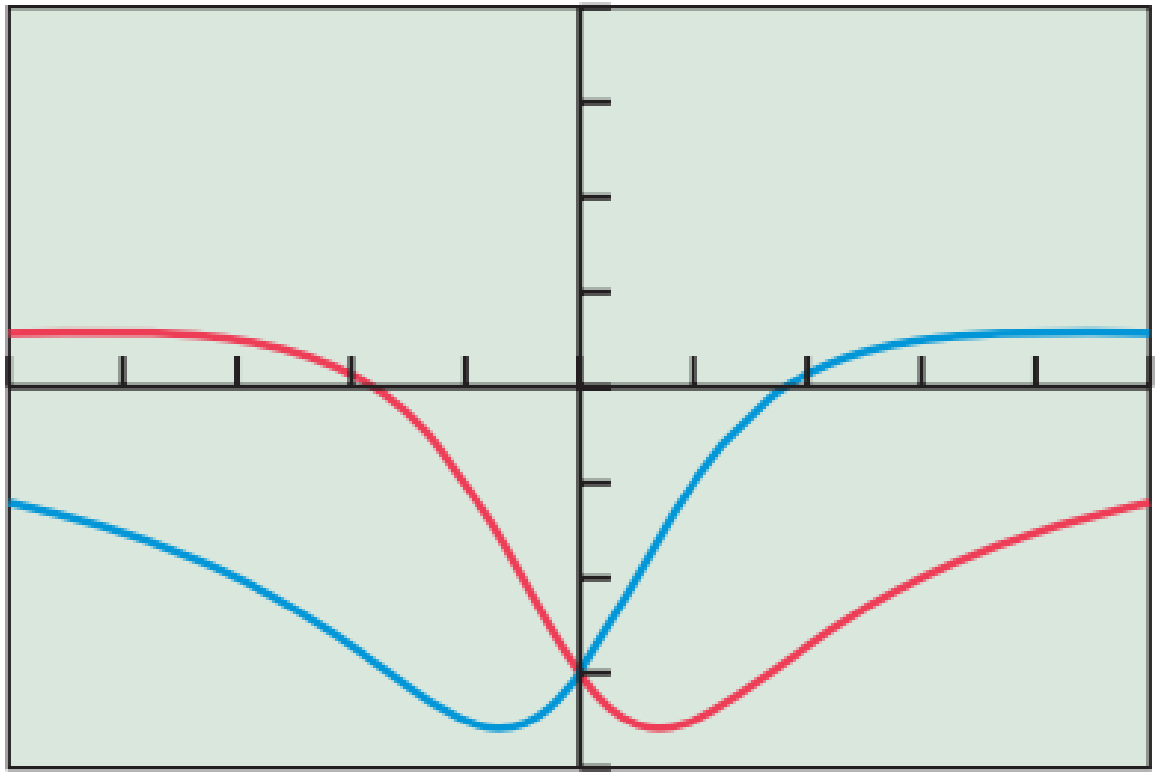
$$y = f(-x)$$

Finding Equations for Reflections

Find an equation for the reflection of $f(x) = \dfrac{5x - 9}{x^2 + 3}$ across each axis.



[–5, 5] by [–4, 4]



[–5, 5] by [–4, 4]

- Open up Desmos
- Login with your google
- Graph the 5 five functions one at a time, taking notice of the differences.
- Describe the changes that you see…..
- Save the file as “*vertical stretch and shrink*”

EXPLORATION 3 Introducing Stretches and Shrinks

Set your viewing window to $[-4.7, 4.7]$ by $[-1.1, 5.1]$ and your graphing mode to sequential as opposed to simultaneous.

1. Graph the functions

$$y_1 = \sqrt{4 - x^2}$$

$$y_2 = 1.5y_1(x) = 1.5\sqrt{4 - x^2}$$

$$y_3 = 2y_1(x) = 2\sqrt{4 - x^2}$$

$$y_4 = 0.5y_1(x) = 0.5\sqrt{4 - x^2}$$

$$y_5 = 0.25y_1(x) = 0.25\sqrt{4 - x^2}$$

on the same screen. What effect do the 1.5, 2, 0.5, and 0.25 seem to have?

- Open a new file
- Repeat the process with the next 5 five functions one at a time, taking notice of the differences.
- Describe the changes that you see in your notes….
- Save the file as *“horizontal stretch and shrink”*

2. Graph the functions

$$y_1 = \sqrt{4 - x^2}$$

$$y_2 = y_1(1.5x) = \sqrt{4 - (1.5x)^2}$$

$$y_3 = y_1(2x) = \sqrt{4 - (2x)^2}$$

$$y_4 = y_1(0.5x) = \sqrt{4 - (0.5x)^2}$$

$$y_5 = y_1(0.25x) = \sqrt{4 - (0.25x)^2}$$

on the same screen. What effect do the 1.5, 2, 0.5, and 0.25 seem to have?

Stretches and Shrinks

Let c be a positive real number. Then the following transformations result in stretches or shrinks of the graph of $y = f(x)$:

Horizontal Stretches or Shrinks

$$y = f\left(\frac{x}{c}\right) \qquad \begin{cases} \text{a stretch by a factor of } c & \text{if } c > 1 \\ \text{a shrink by a factor of } c & \text{if } c < 1 \end{cases}$$

Vertical Stretches or Shrinks

$$y = c \cdot f(x) \qquad \begin{cases} \text{a stretch by a factor of } c & \text{if } c > 1 \\ \text{a shrink by a factor of } c & \text{if } c < 1 \end{cases}$$