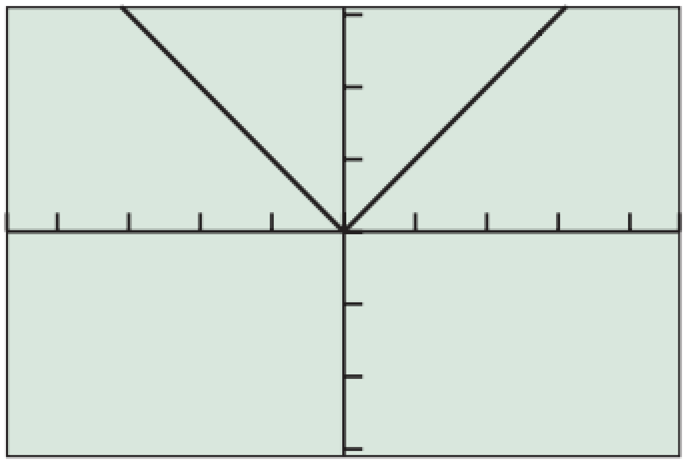
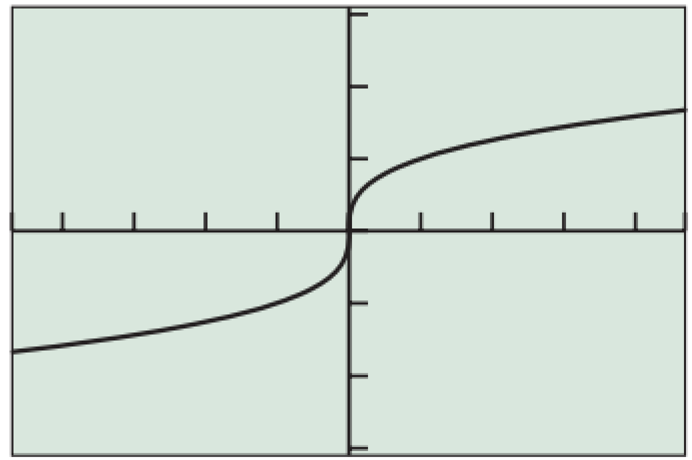
The fact that the derivative of a function at a point can be viewed geometrically as the slope of the line tangent to the curve $y = f(x)$ at that point provides us with some insight as to how a derivative might fail to exist. Unless a function has a well-defined "slope" when you zoom in on it at a, the derivative at a will not exist. For example, Figure 10.3 shows three cases for which $f(0)$ exists but $f'(0)$ does not.



[–4.7, 4.7] by [–3.1, 3.1]

(a)

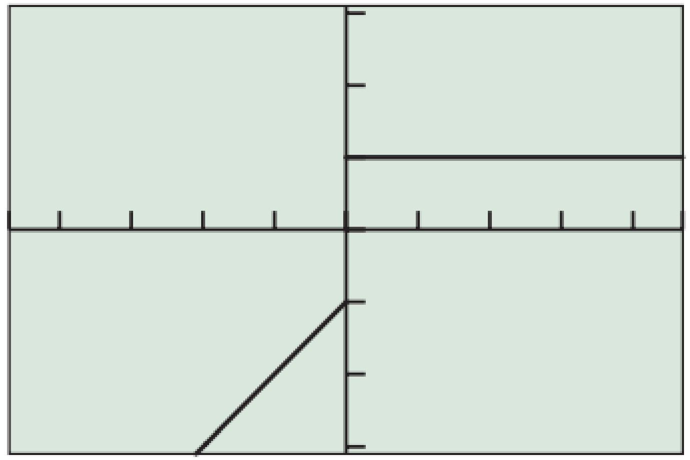
$f(x) = |x|$ has a graph with no definable slope at $x = 0$.



[–4.7, 4.7] by [–3.1, 3.1]

(b)

$f(x) = \sqrt[3]{x}$ has a graph with a vertical tangent line (no slope) at $x = 0$.



[–4.7, 4.7] by [–3.1, 3.1]

(c)

$$f(x) = \begin{cases} x - 1 \text{ for } x < 0 \\ 1 \text{ for } x \geq 0 \end{cases}$$

has a graph with no definable slope at $x = 0$.

EXAMPLE 4 Finding a Derivative at a Point

Find $f'(4)$ if $f(x) = 2x^2 - 3$.

SOLUTION

$$f'(4) = \lim_{h \to 0} \frac{f(4 + h) - f(4)}{h}$$

DEFINITION Derivative

If $y = f(x)$, then the **derivative of the function f with respect to x**, is the function f' whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

for all values of x where the limit exists.

EXAMPLE 5 Finding the Derivative of a Function

(a) Find $f'(x)$ if $f(x) = x^2$.

To emphasize the connection with slope $\Delta y/\Delta x$, Leibniz used the notation dy/dx for the derivative. (The dy and dx were his "ghosts of departed quantities.") This **Leibniz notation** has several advantages over the "prime" notation, as you will learn when you study calculus. We will use both notations in our examples and exercises.

(b) Find $\dfrac{dy}{dx}$ if $y = \dfrac{1}{x}$.