Power Functions and Variation

DEFINITION Power Function

Any function that can be written in the form

$$f(x) = k \cdot x^a, \text{ where } k \text{ and } a \text{ are nonzero constants,}$$

is a **power function**. The constant a is the **power**, and k is the **constant of variation**, or **constant of proportion**. We say $f(x)$ **varies as** the a^{th} power of x, or $f(x)$ **is proportional to** the a^{th} power of x.

Name	Formula	Power	Constant of Variation
Circumference	$C = 2\pi r$	1	2π
Area of a circle	$A = \pi r^2$	2	π
Force of gravity	$F = k/d^2$	-2	k
Boyle's Law	$V = k/P$	-1	k

- The circumference of a circle varies directly as its radius.
- The area enclosed by a circle is directly proportional to the square of its radius.
- The force of gravity acting on an object is inversely proportional to the square of the distance from the object to the center of the Earth.
- Boyle's Law states that the volume of an enclosed gas (at a constant temperature) varies inversely as the applied pressure.

The power function formulas with positive powers are statements of **direct variation** and power function formulas with negative powers are statements of **inverse variation.**

EXAMPLE 1 Writing a Power Function Formula

From empirical evidence and the laws of physics it has been found that the period of time T for the full swing of a pendulum varies as the square root of the pendulum's length l, provided that the swing is small relative to the length of the pendulum.

$$T(l) = k\sqrt{l} \qquad T(l) = k \cdot l^{\frac{1}{2}}$$

Section 1.3 introduced five basic power functions:

$$x,\ x^2,\ x^3,\ x^{-1} = \frac{1}{x},\quad \text{and}\quad x^{1/2} = \sqrt{x}.$$

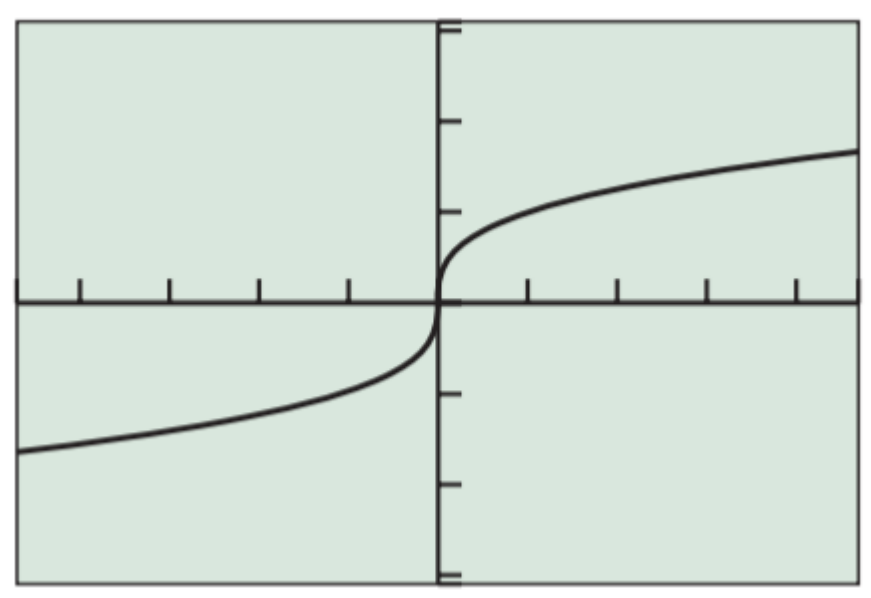
Example 2 describes two other power functions: the *cube root function* and the *inverse-square* function.

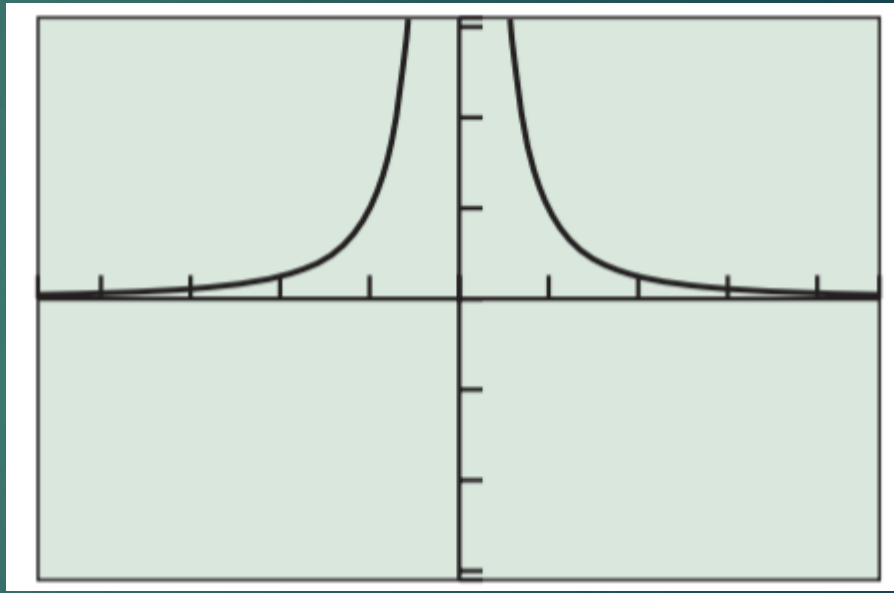
EXAMPLE 2 Analyzing Power Functions

State the power and constant of variation for the function, graph it, and analyze it.

(a) $f(x) = \sqrt[3]{x}$

(b) $g(x) = \dfrac{1}{x^2}$



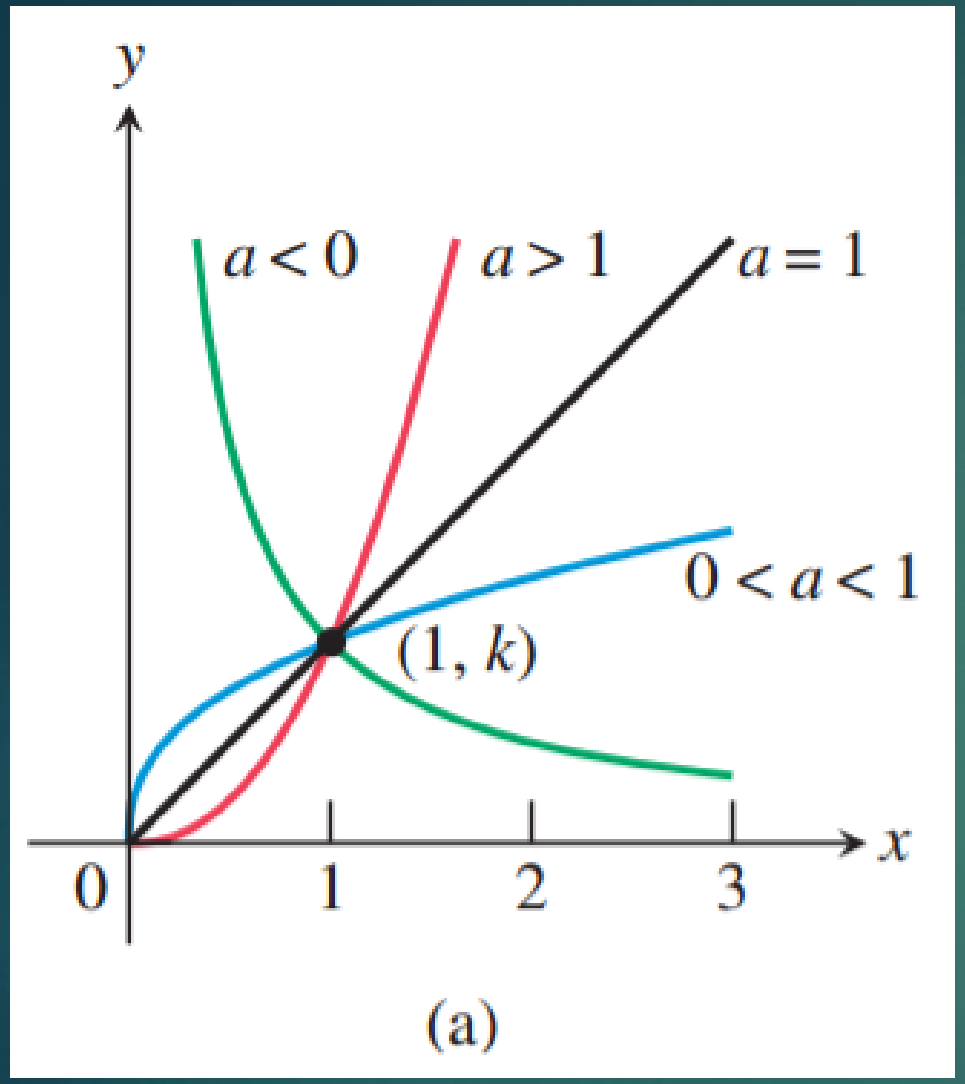


DEFINITION Monomial Function

Any function that can be written as

$$f(x) = k \text{ or } f(x) = k \cdot x^n,$$

where k is a constant and n is a positive integer, is a **monomial function**.



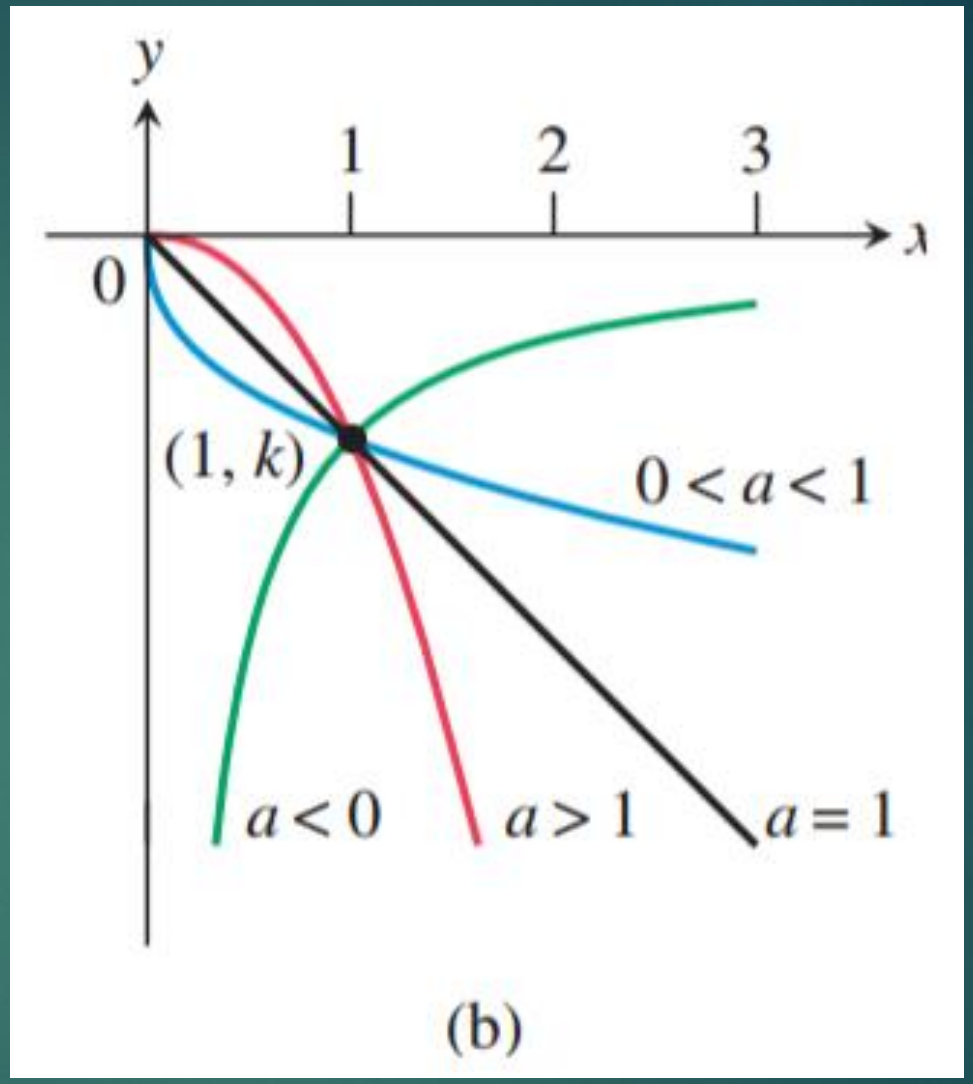


FIGURE 2.14 The graphs of $f(x) = k \cdot x^a$ for $x \geq 0$. (a) $k > 0$, (b) $k < 0$.

Table 2.10 Average Distances and Orbit Periods for the Six Innermost Planets

Planet	Average Distance from Sun (Gm)	Period of Orbit (days)
Mercury	57.9	88
Venus	108.2	225
Earth	149.6	365.2
Mars	227.9	687
Jupiter	778.3	4332
Saturn	1427	10,760

Source: Shupe, Dorr, Payne, Hunsiker, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

EXAMPLE 5 Modeling Planetary Data with a Power Function

Use the data in Table 2.10 to obtain a power function model for orbital period as a function of average distance from the Sun. Then use the model to predict the orbital period for Neptune, which is 4497 Gm from the Sun on average.