Polynomial Functions of Higher Degree with Modeling

DEFINITION The Vocabulary of Polynomials

- Each monomial in this sum—$a_n x^n$, $a_{n-1} x^{n-1}, \ldots, a_0$—is a **term** of the polynomial.
- A polynomial function written in this way, with terms in descending degree, is written in **standard form**.
- The constants $a_n, a_{n-1}, \ldots, a_0$ are the **coefficients** of the polynomial.
- The term $a_n x^n$ is the **leading term**, and a_0 is the constant term.

Graphing Transformations of Monomial Functions

(a) $g(x) = 4(x + 1)^3$

(b) $h(x) = -(x - 2)^4 + 5$

THEOREM Local Extrema and Zeros of Polynomial Functions

A polynomial function of degree n has at most $n - 1$ local extrema and at most n zeros.

End Behavior of Polynomial Functions

EXPLORATION 1 **Investigating the End Behavior of $f(x) = a_n x^n$**

Graph each function in the window $[-5, 5]$ by $[-15, 15]$. Describe the end behavior using $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

1. **(a)** $f(x) = 2x^3$ **(b)** $f(x) = -x^3$
(c) $f(x) = x^5$ **(d)** $f(x) = -0.5x^7$

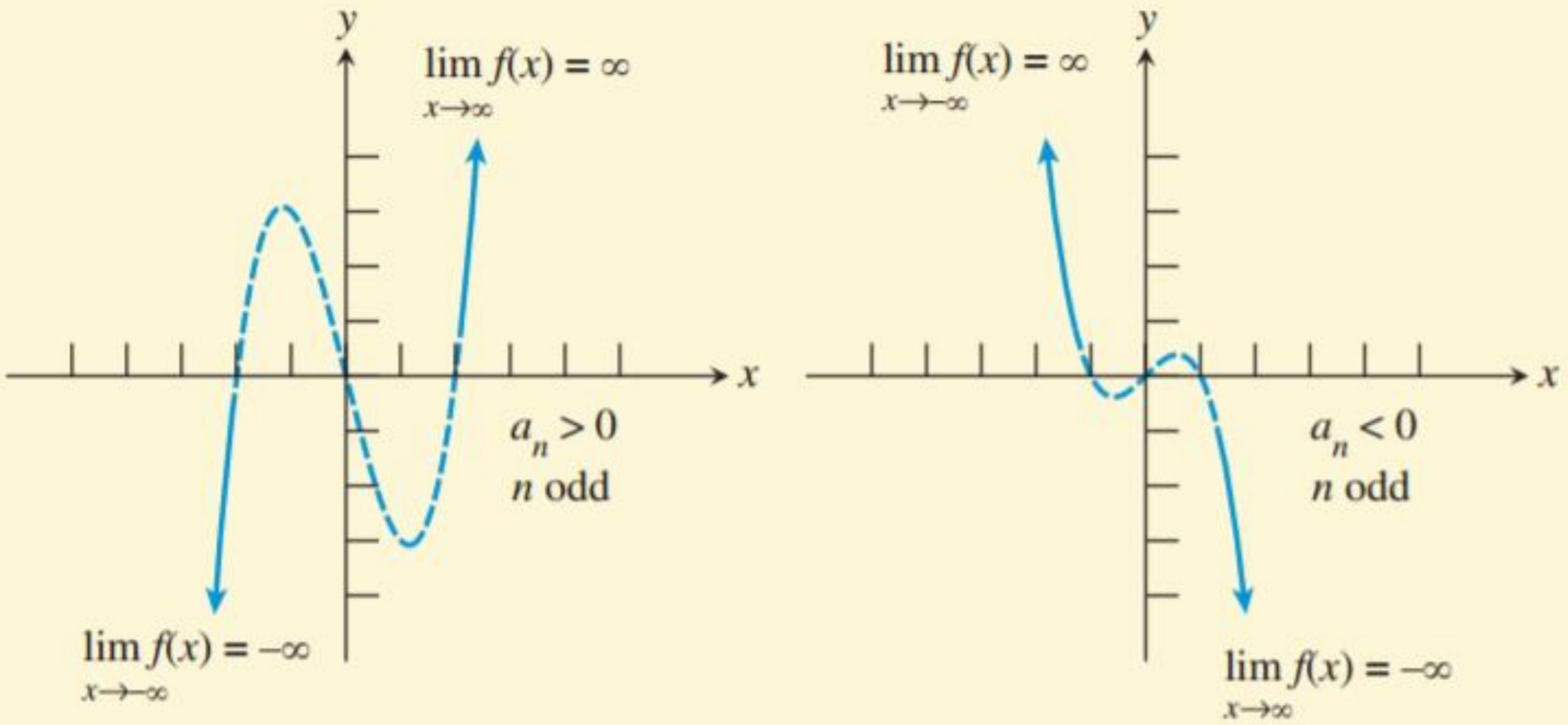
2. **(a)** $f(x) = -3x^4$ **(b)** $f(x) = 0.6x^4$
(c) $f(x) = 2x^6$ **(d)** $f(x) = -0.5x^2$

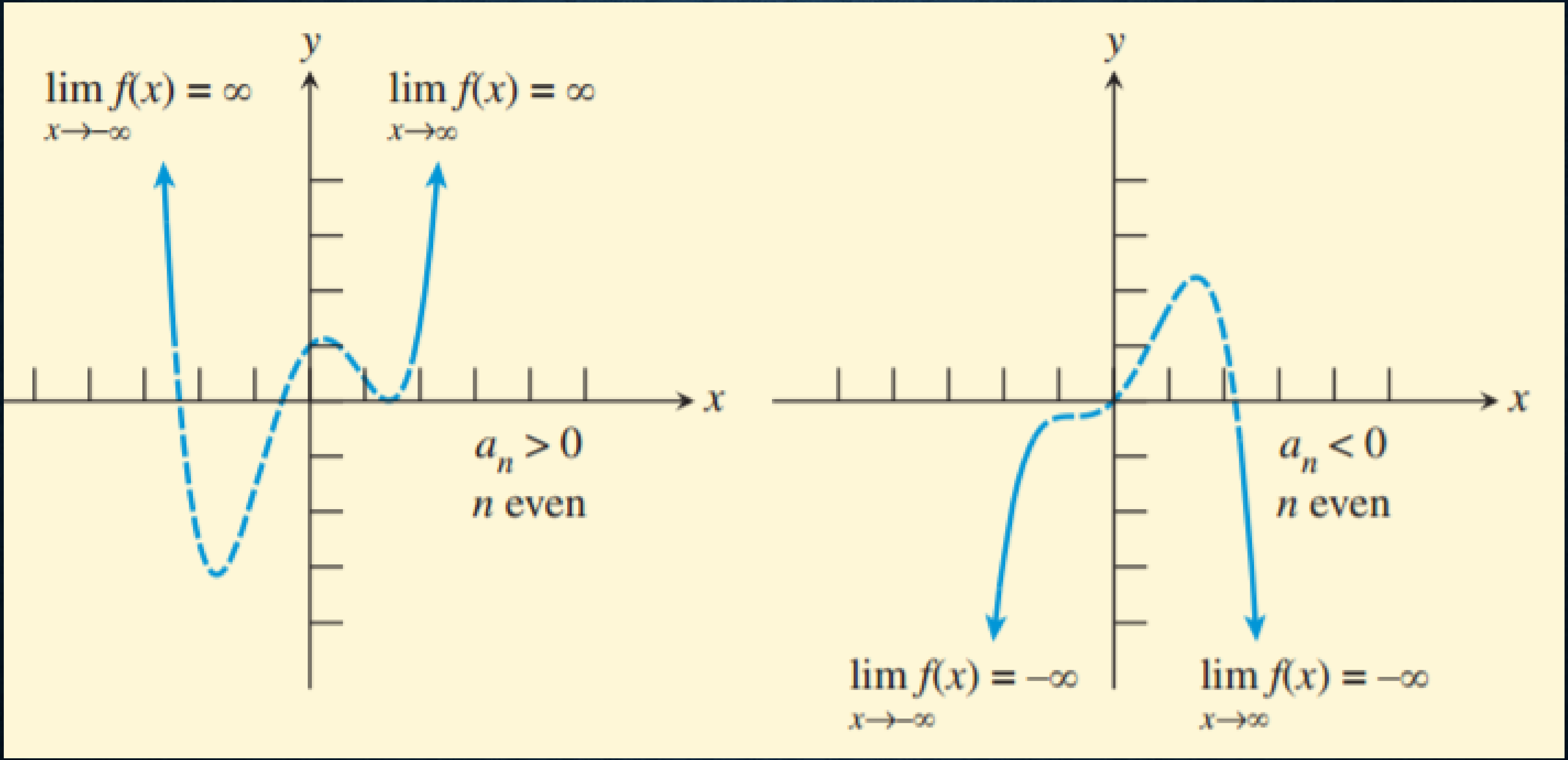
3. **(a)** $f(x) = -0.3x^5$ **(b)** $f(x) = -2x^2$
(c) $f(x) = 3x^4$ **(d)** $f(x) = 2.5x^3$

Describe the patterns you observe. In particular, how do the values of the coefficient a_n and the degree n affect the end behavior of $f(x) = a_n x^n$?

Leading Term Test for Polynomial End Behavior

For any polynomial function $f(x) = a_n x^n + \cdots + a_1 x + a_0$, the limits $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ are determined by the degree n of the polynomial and its leading coefficient a_n:





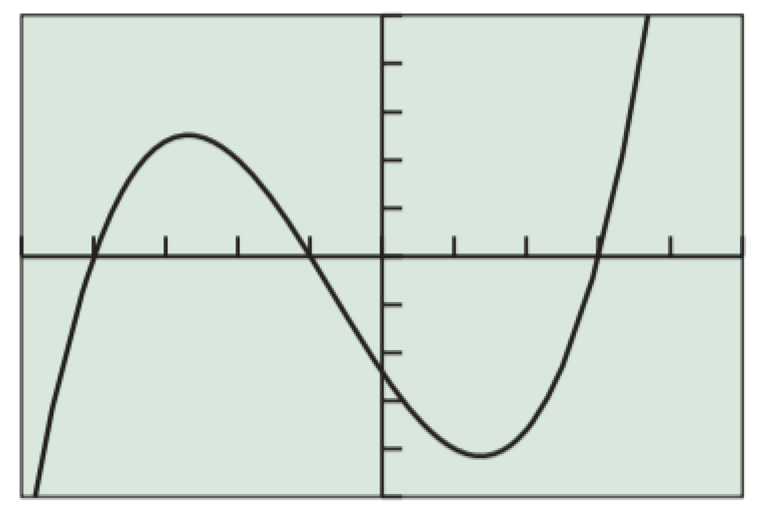
$\lim_{x \to -\infty} f(x) = \infty$
$\lim_{x \to \infty} f(x) = \infty$
y
x
$a_n > 0$
n even
y
x
$a_n < 0$
n even
$\lim_{x \to -\infty} f(x) = -\infty$
$\lim_{x \to \infty} f(x) = -\infty$

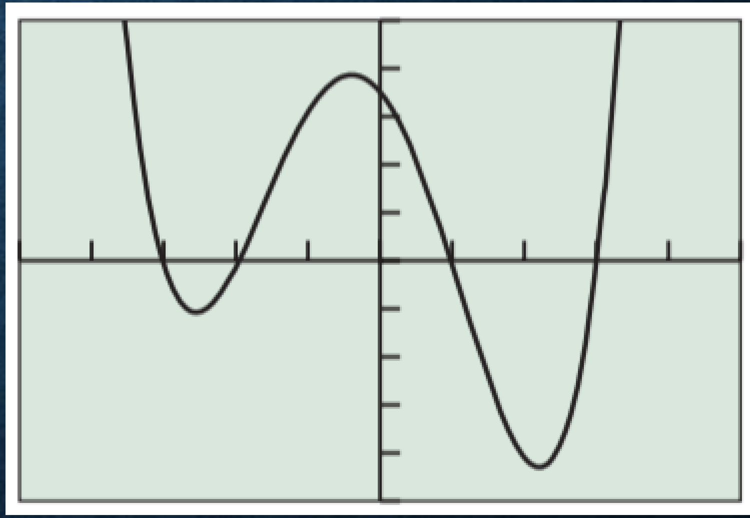
EXAMPLE 4 **Applying Polynomial Theory**

Graph the polynomial in a window showing its extrema and zeros and its end behavior. Describe the end behavior using limits.

(a) $f(x) = x^3 + 2x^2 - 11x - 12$

(b) $g(x) = 2x^4 + 2x^3 - 22x^2 - 18x + 35$





EXAMPLE 5 Finding the Zeros of a Polynomial Function

Find the zeros of $f(x) = x^3 - x^2 - 6x$.

DEFINITION Multiplicity of a Zero of a Polynomial Function

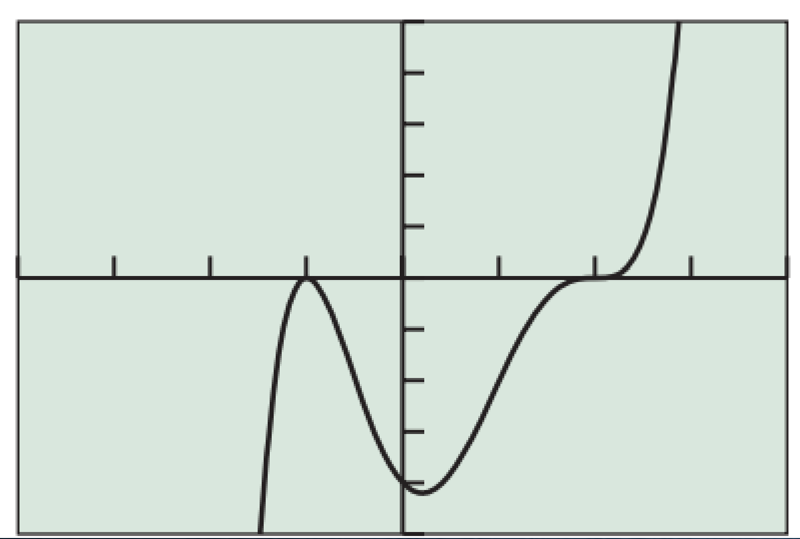


FIGURE 2.27 The graph of $f(x) = (x - 2)^3(x + 1)^2$ showing the x-intercepts.

repeated zero.

Zeros of Odd and Even Multiplicity

If a polynomial function f has a real zero c of odd multiplicity, then the graph of f crosses the x-axis at $(c, 0)$ and the value of f changes sign at $x = c$.

If a polynomial function f has a real zero c of even multiplicity, then the graph of f does not cross the x-axis at $(c, 0)$ and the value of f does not change sign at $x = c$.

EXAMPLE 6 Sketching the Graph of a Factored Polynomial

State the degree and list the zeros of the function $f(x) = (x + 2)^3(x - 1)^2$. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of f by hand.