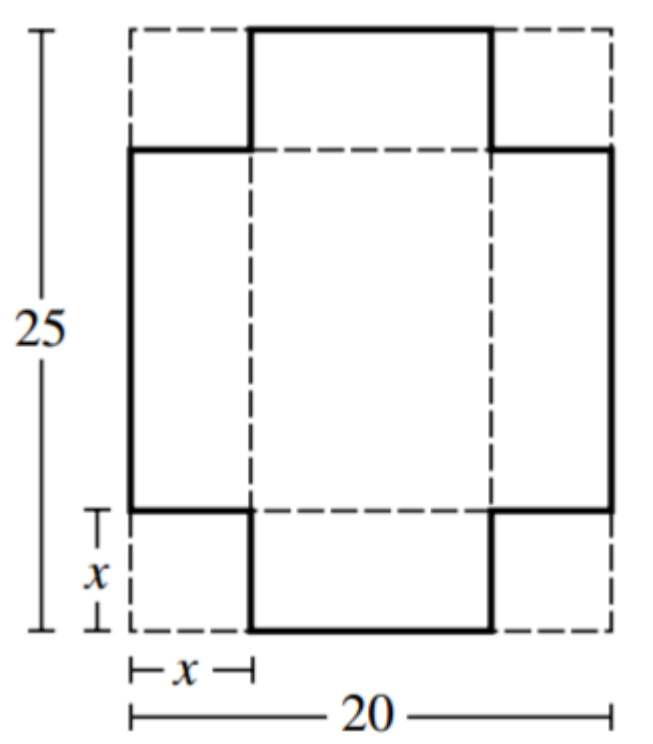
EXAMPLE 9 Designing a Box

Dixie Packaging Company has contracted to make boxes with a volume of approximately 484 in.^{3}. Squares are to be cut from the corners of a 20-in. by 25-in. piece of cardboard, and the flaps folded up to make an open box. (See Figure 2.31.) What size squares should be cut from the cardboard?



THEOREM Intermediate Value Theorem

If a and b are real numbers with $a < b$ and if f is continuous on the interval $[a, b]$, then f takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some number c in $[a, b]$.

In particular, if $f(a)$ and $f(b)$ have opposite signs (i.e., one is negative and the other is positive), then $f(c) = 0$ for some number c in $[a, b]$ (Figure 2.29).

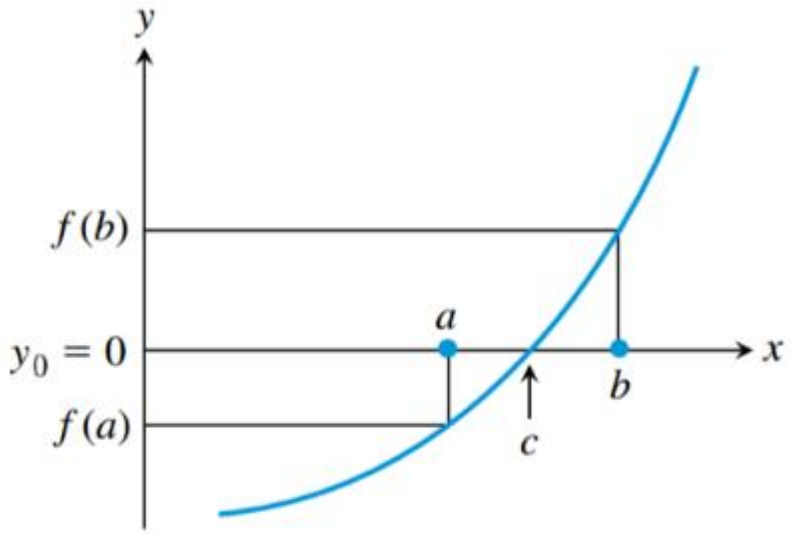


FIGURE 2.29 If $f(a) < 0 < f(b)$, then there is a zero $x = c$ between a and b.

In Exercises 17–24, graph the function in a viewing window that shows all of its extrema and x-intercepts. Describe the end behavior using limits.

18. $f(x) = (2x - 3)(4 - x)(x + 1)$

In Exercises 39–42, state the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of the polynomial function by hand.

42. $f(x) = 7(x - 3)^2(x + 5)^4$