## EXAMPLE 4 Finding the Rational Zeros

Find the rational zeros of $f(x) = x^3 - 3x^2 + 1$.

# Upper and Lower Bounds

## Upper and Lower Bound Tests for Real Zeros

Let $f$ be a polynomial function of degree $n \geq 1$ with a positive leading coefficient. Suppose $f(x)$ is divided by $x - k$ using synthetic division.

- If $k \geq 0$ and every number in the last line is nonnegative (positive or zero), then $k$ is an *upper bound* for the real zeros of $f$.
- If $k \leq 0$ and the numbers in the last line are alternately nonnegative and non-positive, then $k$ is a *lower bound* for the real zeros of $f$.

## EXAMPLE 6 Establishing Bounds for Real Zeros

Prove that all of the real zeros of $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$ must lie in the interval $[-2, 5]$.

## EXAMPLE 7 Finding the Real Zeros of a Polynomial Function

Find all of the real zeros of $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$.

## EXAMPLE 8 Finding the Real Zeros of a Polynomial Function

Prove that all of the real zeros of $f(x) = 10x^5 - 3x^2 + x - 6$ lie in the interval $[0, 1]$, and find them.