# Exponential and Logistic Functions

## Exponential Functions and Their Graphs

**DEFINITION Exponential Functions**

Let $a$ and $b$ be real number constants. An **exponential function** in $x$ is a function that can be written in the form

$$f(x) = a \cdot b^x,$$

where $a$ is nonzero, $b$ is positive, and $b \neq 1$. The constant $a$ is the *initial value* of $f$ (the value at $x = 0$), and $b$ is the **base**.

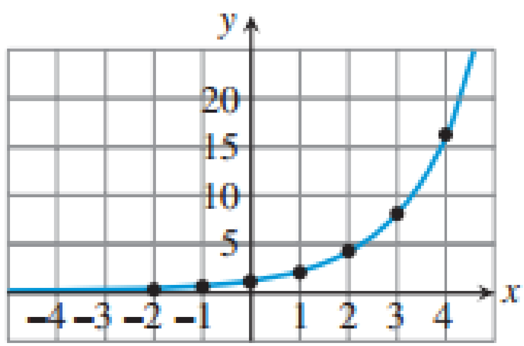



**FIGURE 3.1** Sketch of $g(x) = 2^x$.

## EXAMPLE 2 Computing Exponential Function Values for Rational Number Inputs

For $f(x) = 2^x$,

**(a)** $f(4) = 2^4$

**(b)** $f(0) = 2^0 =$

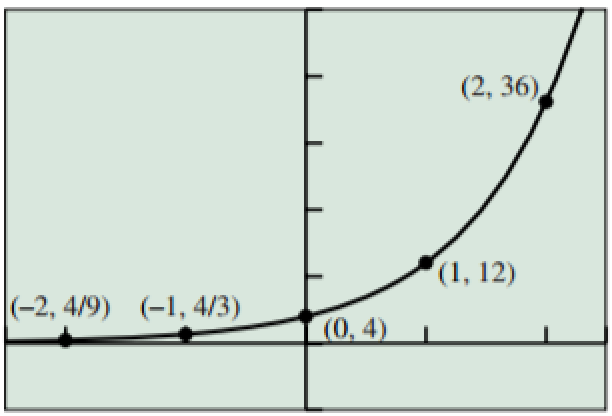
**(c)** $f(-3) = 2^{-3} =$

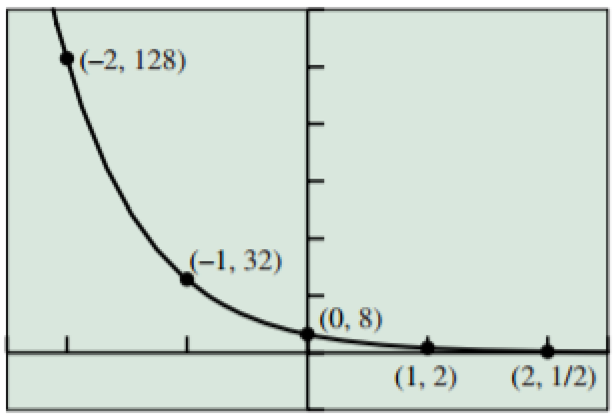
**(d)** $f\left(\frac{1}{2}\right) = 2^{1/2} =$

**(e)** $f\left(-\frac{3}{2}\right) = 2^{-3/2} =$

## Table 3.2 Values for Two Exponential Functions

| $x$ | $g(x)$ | $h(x)$ |
|---|---|---|
| $-2$ | $4/9$ | 128 |
| $-1$ | $4/3$ | 32 |
| 0 | 4 | 8 |
| 1 | 12 | 2 |
| 2 | 36 | $1/2$ |



## Exponential Growth and Decay

For any exponential function $f(x) = a \cdot b^x$ and any real number $x$,

If $a > 0$ and $b > 1$, the function $f$ is increasing and is an **exponential growth function**. The base $b$ is its **growth factor**.

If $a > 0$ and $b < 1$, $f$ is decreasing and is an **exponential decay function**. The base $b$ is its **decay factor**.

## EXAMPLE 4 Transforming Exponential Functions

Describe how to transform the graph of $f(x) = 2^x$ into the graph of the given function. Sketch the graphs by hand and support your answer with a grapher.

**(a)** $g(x) = 2^{x-1}$ **(b)** $h(x) = 2^{-x}$ **(c)** $k(x) = 3 \cdot 2^x$

## DEFINITION The Natural Base *e*

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$