Constant Percentage Rate and Exponential Functions

Suppose that a population is changing at a **constant percentage rate *r***, where r is the percent rate of change expressed in decimal form. Then the population follows the pattern shown.

Time in years	Population
0	$P(0) = P_0 =$ initial population
1	$P(1) = P_0 + P_0 r = P_0(1 + r)$
2	$P(2) = P(1) \cdot (1 + r) = P_0(1 + r)^2$
3	$P(3) = P(2) \cdot (1 + r) = P_0(1 + r)^3$
$\vdots$	$\vdots$
t	$P(t) = P_0(1 + r)^t$

Exponential Population Model

If a population P is changing at a constant percentage rate r each year, then

$$P(t) = P_0(1 + r)^t,$$

where P_0 is the initial population, r is expressed as a decimal, and t is time in years.

If $r > 0$, then $P(t)$ is an exponential growth function, and its *growth factor* is the base of the exponential function, $1 + r$.

On the other hand, if $r < 0$, the base $1 + r < 1$, $P(t)$ is an exponential decay function, and $1 + r$ is the *decay factor* for the population.

EXAMPLE 1 Finding Growth and Decay Rates

Tell whether the population model is an exponential growth function or exponential decay function, and find the constant percentage rate of growth or decay.

(a) San Jose: $P(t) = 782{,}248 \cdot 1.0136^t$

(b) Detroit: $P(t) = 1{,}203{,}368 \cdot 0.9858^t$

EXAMPLE 2 Finding an Exponential Function

Determine the exponential function with initial value $= 12$, increasing at a rate of 8% per year.

EXAMPLE 3 Modeling Bacteria Growth

Suppose a culture of 100 bacteria is put into a petri dish and the culture doubles every hour. Predict when the number of bacteria will be 350,000.

EXAMPLE 4 Modeling Radioactive Decay

Suppose the half-life of a certain radioactive substance is 20 days and there are 5 g (grams) present initially. Find the time when there will be 1 g of the substance remaining.

Scientists have established that atmospheric pressure at sea level is 14.7 lb/in.^{2}, and the pressure is reduced by half for each 3.6 mi above sea level. For example, the pressure 3.6 mi above sea level is $(1/2)(14.7) = 7.35$ lb/in.^{2}. This rule for atmospheric pressure holds for altitudes up to 50 mi above sea level. Though the context is different, the mathematics of atmospheric pressure closely resembles the mathematics of radioactive decay.

EXAMPLE 5 Determining Altitude from Atmospheric Pressure

Find the altitude above sea level at which the atmospheric pressure is 4 lb/in.^{2}.

EXAMPLE 6 Modeling U.S. Population Using Exponential Regression

Use the 1900–2000 data in Table 3.9 and exponential regression to predict the U.S. population for 2003. Compare the result with the listed value for 2003.

Table 3.9 U.S. Population (in millions)

Year	Population
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.2
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7
2000	281.4
2003	290.8