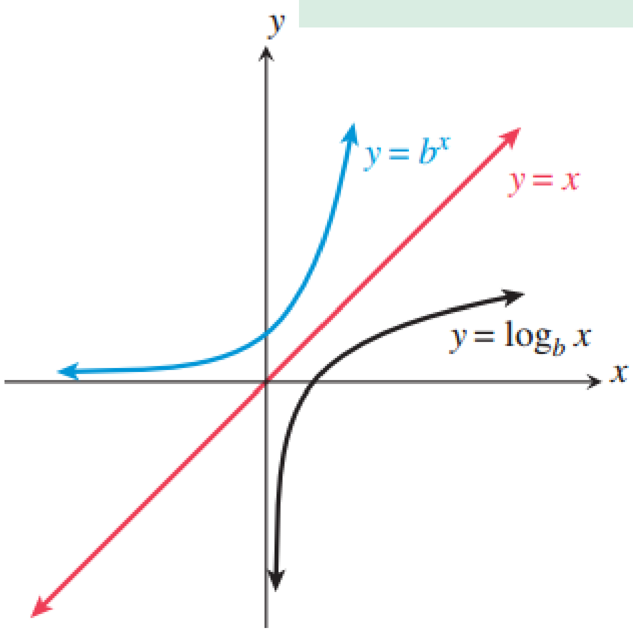
Logarithmic Functions and Their Graphs

Inverses of Exponential Functions

Changing Between Logarithmic and Exponential Form

If $x > 0$ and $0 < b \neq 1$, then

$$y = \log_b(x) \quad \text{if and only if} \quad b^y = x.$$



EXAMPLE 1 Evaluating Logarithms

(a) $\log_2 8 =$

(b) $\log_3 \sqrt{3} =$

(c) $\log_5 \dfrac{1}{25} =$

(d) $\log_4 1 =$

(e) $\log_7 7 =$

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

EXAMPLE 2 Evaluating Logarithmic and Exponential Expressions

(a) $\log_2 8 =$

(b) $\log_3 \sqrt{3} =$

(c) $6^{\log_6 11} =$

Common Logarithms—Base 10

Basic Properties of Common Logarithms

Let x and y be real numbers with $x > 0$.

- $\log 1 = 0$ because $10^0 = 1$.
- $\log 10 = 1$ because $10^1 = 10$.
- $\log 10^y = y$ because $10^y = 10^y$.
- $10^{\log x} = x$ because $\log x = \log x$.

EXAMPLE 3 Evaluating Logarithmic and Exponential Expressions—Base 10

(a) $\log 100 =$

(b) $\log \sqrt[5]{10} =$

(c) $\log \dfrac{1}{1000} =$

(d) $10^{\log 6} =$

EXAMPLE 5 Solving Simple Logarithmic Equations

Solve each equation by changing it to exponential form.

(a) $\log x = 3$ **(b)** $\log_2 x = 5$

Basic Properties of Natural Logarithms

Let x and y be real numbers with $x > 0$.

- $\ln 1 = 0$ because $e^0 = 1$.
- $\ln e = 1$ because $e^1 = e$.
- $\ln e^y = y$ because $e^y = e^y$.
- $e^{\ln x} = x$ because $\ln x = \ln x$.

EXAMPLE 6 Evaluating Logarithmic and Exponential Expressions—Base *e*

(a) $\ln \sqrt{e} =$

(b) $\ln e^5 =$

(c) $e^{\ln 4} =$