EXPLORATION 2 Discovering Relationships and Nonrelationships

Of the eight relationships suggested here, four are *true* and four are *false* (using values of x within the domains of both sides of the equations). Thinking about the properties of logarithms, make a prediction about the truth of each statement. Then test each with some specific numerical values for x. Finally, compare the graphs of the two sides of the equation.

1. $\ln (x + 2) = \ln x + \ln 2$

2. $\log_3 (7x) = 7 \log_3 x$

3. $\log_2 (5x) = \log_2 5 + \log_2 x$

4. $\ln \dfrac{x}{5} = \ln x - \ln 5$

5. $\log \dfrac{x}{4} = \dfrac{\log x}{\log 4}$

6. $\log_4 x^3 = 3 \log_4 x$

7. $\log_5 x^2 = (\log_5 x)(\log_5 x)$

8. $\log |4x| = \log 4 + \log |x|$

Which four are true, and which four are false?

Properties of Logarithms

Let b, R, and S be positive real numbers with $b \neq 1$, and c any real number.

- **Product rule**: $\log_b (RS) = \log_b R + \log_b S$
- **Quotient rule**: $\log_b \frac{R}{S} = \log_b R - \log_b S$
- **Power rule**: $\log_b R^c = c \log_b R$

EXAMPLE 2 Expanding the Logarithm of a Product

Assuming x and y are positive, use properties of logarithms to write $\log (8xy^4)$ as a sum of logarithms or multiples of logarithms.

$$\log (8xy^4) = \log 8 + \log x + \log y^4$$

$$= \log 2^3 + \log x + \log y^4$$

$$= 3 \log 2 + \log x + 4 \log y$$

EXAMPLE 4 Condensing a Logarithmic Expression

Assuming x and y are positive, write $\ln x^5 - 2 \ln (xy)$ as a single logarithm.

$$\ln x^5 - \ln (xy)^2$$

$$\ln x^5 - \ln (x^2y^2)$$

$$\ln \frac{x^5}{x^2 y^2}$$

$$\ln \frac{x^3}{y^2}$$

Change-of-Base Formula for Logarithms

For positive real numbers a, b, and x with $a \neq 1$ and $b \neq 1$,

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

$$\log_4 7 = \frac{\ln 7}{\ln 4} =$$

EXAMPLE 5 Evaluating Logarithms by Changing the Base

(a) $\log_3 16 =$

(b) $\log_6 10 =$

(c) $\log_{1/2} 2 =$

EXAMPLE 6 Graphing logarithmic functions

Describe how to transform the graph of $f(x) = \ln x$ into the graph of the given function. Sketch the graph by hand and support your answer with a grapher.

(a) $g(x) = \log_5 x$

(b) $h(x) = \log_{1/4} x$