One-to-One Properties

For any exponential function $f(x) = b^x$,

- If $b^u = b^v$, then $u = v$.

For any logarithmic function $f(x) = \log_b x$,

- If $\log_b u = \log_b v$, then $u = v$.

EXAMPLE 1 Solving an Exponential Equation Algebraically

Solve $20(1/2)^{x/3} = 5$.

EXAMPLE 3 Solving a Logarithmic Equation

Solve $\log x^2 = 2$.

EXAMPLE 4 Solving a Logarithmic Equation

Solve $\ln(3x - 2) + \ln(x - 1) = 2 \ln x$.

EXAMPLE 5 Comparing Earthquake Intensities

How many times more severe was the 2001 earthquake in Gujarat, India ($R_1 = 7.9$) than the 1999 earthquake in Athens, Greece ($R_2 = 5.9$)?

$$\mathrm{pH} = -\log[\mathrm{H}^+]$$

EXAMPLE 6 Comparing Chemical Acidity

Some especially sour vinegar has a pH of 2.4, and a box of Leg and Sickle baking soda has a pH of 8.4.

(a) What are their hydrogen-ion concentrations?

(b) How many times greater is the hydrogen-ion concentration of the vinegar than that of the baking soda?

(c) By how many orders of magnitude do the concentrations differ?

Newton's Law of Cooling

An object that has been heated will cool to the temperature of the medium in which it is placed, such as the surrounding air or water. The temperature T of the object at time t can be modeled by

$$T(t) = T_m + (T_0 - T_m)e^{-kt}$$

for an appropriate value of k, where

T_m = the temperature of the surrounding medium,

T_0 = initial temperature of the object.

EXAMPLE 7 Applying Newton's Law of Cooling

A hard-boiled egg at temperature 96°C is placed in 16°C water to cool. Four minutes later the temperature of the egg is 45°C. Use Newton's Law of Cooling to determine when the egg will be 20°C.

SOLUTION

Model Because $T_0 = 96$ and $T_m = 16$, $T_0 - T_m = 80$ and

$$T(t) = T_m + (T_0 - T_m)e^{-kt} = 16 + 80e^{-kt}.$$