Interest Compounded Annually

If a principal P is invested at a fixed annual interest rate r, calculated at the end of each year, then the value of the investment after n years is

$$A = P(1 + r)^n,$$

where r is expressed as a decimal.

EXAMPLE 1 Compounding Annually

Suppose Quan Li invests \$500 at 7% interest compounded annually. Find the value of her investment 10 years later.

$$A = P(1+r)^t$$

$$A = 500(1+.07)^{10}$$

$$A = \$983.58$$

Interest Compounded k Times per Year

Suppose a principal P is invested at an annual interest rate r compounded k times a year for t years. Then r/k is the interest rate per compounding period, and kt is the number of compounding periods. The amount A in the account after t years is

$$A = P\left(1 + \frac{r}{k}\right)^{kt}.$$

EXAMPLE 2 Compounding Monthly

Suppose Roberto invests \$500 at 9% annual interest *compounded monthly*, that is, compounded 12 times a year. Find the value of his investment 5 years later.

$$A = 500\left(1 + \frac{0.09}{12}\right)^{12(5)}$$

$$A = \$782.84$$

EXAMPLE 3 Finding the Time Period of an Investment

Judy has \$500 to invest at 9% annual interest compounded monthly. How long will it take for her investment to grow to \$3000?

$$3000 = 500\left(1+\frac{0.09}{12}\right)^{12(t)}$$

$$\log 6 = (12t)\log\left(1+\frac{0.09}{12}\right)$$

$$\frac{\log 6}{12\log\left(1+\frac{0.09}{12}\right)} = t$$

$$t \approx 19.98 \text{ years}$$

EXAMPLE 4 Finding an Interest Rate

Stephen has \$500 to invest. What annual interest rate *compounded quarterly* (four times per year) is required to double his money in 10 years?

$$2 = \left(1 + \frac{i}{4}\right)^{4(10)}$$

$$2 = \left(1 + \frac{i}{4}\right)^{40}$$

$$\sqrt[40]{2} = 1 + \frac{i}{4}$$

$$\sqrt[40]{2} - 1 = \frac{i}{4}$$

$$4\left(\sqrt[40]{2} - 1\right) = i$$

$$i = 0.0699187$$

$$i \approx 6.99\%$$

Compound Interest—Value of an Investment

Suppose a principal P is invested at a fixed annual interest rate r. The value of the investment after t years is

- $A = P\left(1 + \frac{r}{k}\right)^{kt}$ when interest compounds k times per year,
- $A = Pe^{rt}$ when interest compounds continuously.

EXAMPLE 5 Compounding Continuously

Suppose LaTasha invests \$100 at 8% annual interest compounded continuously. Find the value of her investment at the end of each of the years 1, 2, . . . , 7.

$$A = 100e^{(.08)1} = \$108.33$$

$$A = 100e^{(.08)2} = \$117.35$$

$$A = 100e^{(.08)7} = \$175.07$$

EXAMPLE 7 Comparing Annual Percentage Yields (APYs)

Which investment is more attractive, one that pays 8.75% compounded quarterly or another that pays 8.7% compounded monthly?

$$1 + r_{effective} = \left(1 + \frac{r}{n}\right)^n$$

$$r_{effective} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$r_{effective} = \left(1 + \frac{.0875}{4}\right)^4 - 1$$

$$r_{effective} \approx 9.04\%$$

$$r_{effective} = \left(1 + \frac{.087}{12}\right)^{12} - 1$$

$$r_{effective} \approx 9.05\%$$

8.7% compounded monthly is slightly better than 8.75% compounded quarterly

EXAMPLE 6 Computing Annual Percentage Yield (APY)

Ursula invests \$2000 with Crab Key Bank at 5.15% annual interest compounded quarterly. What is the equivalent APY?

$$r_{effective} = \left(1 + \frac{.0515}{4}\right)^4 - 1$$

$$r_{effective} \approx 5.25\%$$

Future Value of an Annuity

The future value FV of an annuity consisting of n equal periodic payments of R dollars at an interest rate i per compounding period (payment interval) is

$$FV = R\frac{(1+i)^n - 1}{i}.$$

$$FV(i) = R\left[(1+i)^n - 1\right]$$

$$\frac{FV(i)}{\left[(1+i)^n - 1\right]} = R$$

EXAMPLE 8 Calculating the Value of an Annuity

At the end of each quarter year, Emily makes a \$500 payment into the Lanaghan Mutual Fund. If her investments earn 7.88% annual interest compounded quarterly, what will be the value of Emily's annuity in 20 years?

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$FV = \frac{500\left[(1+0.0788/4)^{80} - 1\right]}{(0.0788/4)}$$

Present Value of an Annuity

The present value PV of an annuity consisting of n equal payments of R dollars earning an interest rate i per period (payment interval) is

$$PV = R\frac{1-(1+i)^{-n}}{i}.$$

EXAMPLE 9 Calculating Loan Payments

Carlos purchases a new pickup truck for \$18,500. What are the monthly payments for a 4-year loan with a \$2000 down payment if the annual interest rate (APR) is 2.9%?

$$PV(i) = R\left[1-(1+i)^{-n}\right]$$

$$\frac{PV(i)}{\left[1-(1+i)^{-n}\right]} = R$$

$$\frac{16500(0.029/12)}{\left[1-(1+0.029/12)^{-48}\right]} = R$$