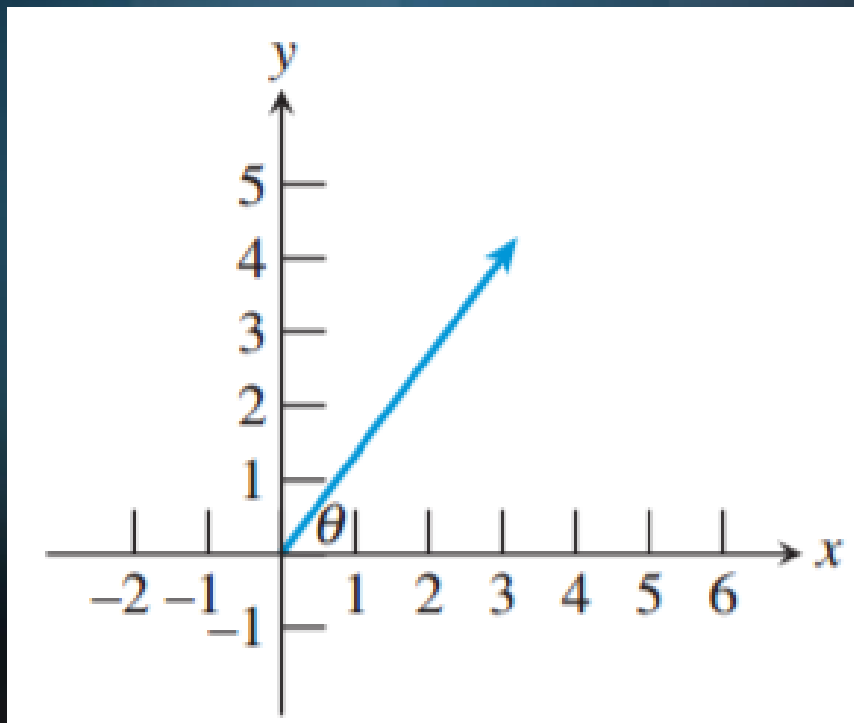
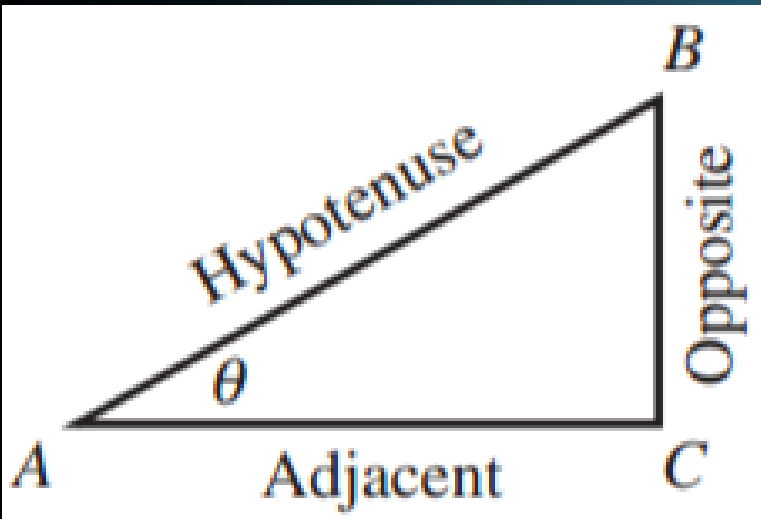


4.2

Trigonometric Functions of Acute Angles



To bring the power of coordinate geometry into the picture, we will often put our acute angles in **standard position** in the xy -plane, with the vertex at the origin, one ray along the positive x -axis, and the other ray extending into the first quadrant. (See Figure 4.7.)



DEFINITION Trigonometric Functions

Let θ be an acute angle in the right $\triangle ABC$ (Figure 4.8). Then

$$\text{sine } (\theta) = \sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\text{cosecant } (\theta) = \csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\text{cosine } (\theta) = \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\text{secant } (\theta) = \sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\text{tangent } (\theta) = \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\text{cotangent } (\theta) = \cot \theta = \frac{\text{adj}}{\text{opp}}$$

EXAMPLE 1 Evaluating Trigonometric Functions of 45°

Find the values of all six trigonometric functions for an angle of 45° .

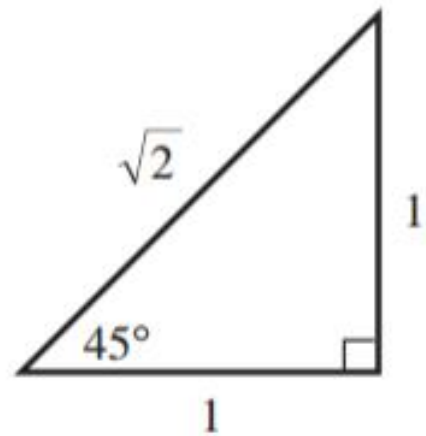


FIGURE 4.9 An isosceles right triangle.
(Example 1)

$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\cos 45^\circ = \frac{\text{adj}}{\text{hyp}}$$

$$\tan 45^\circ = \frac{\text{opp}}{\text{adj}}$$

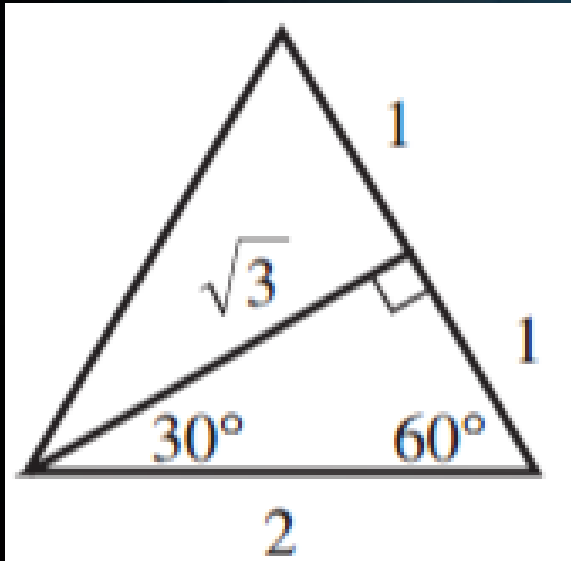
$$\csc 45^\circ = \frac{\text{hyp}}{\text{opp}}$$

$$\sec 45^\circ = \frac{\text{hyp}}{\text{adj}}$$

$$\cot 45^\circ = \frac{\text{adj}}{\text{opp}}$$

EXAMPLE 2 Evaluating Trigonometric Functions of 30°

Find the values of all six trigonometric functions for an angle of 30° .



$$\sin 30^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\cos 30^\circ = \frac{\text{adj}}{\text{hyp}}$$

$$\tan 30^\circ = \frac{\text{opp}}{\text{adj}}$$

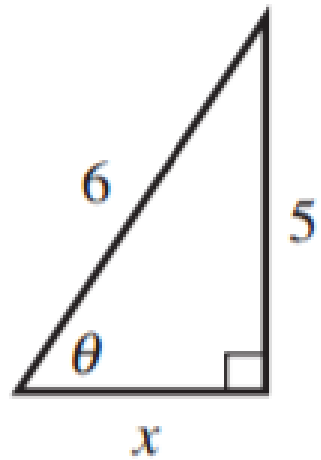
$$\csc 30^\circ = \frac{\text{hyp}}{\text{opp}}$$

$$\sec 30^\circ = \frac{\text{hyp}}{\text{adj}}$$

$$\cot 30^\circ = \frac{\text{adj}}{\text{opp}}$$

EXAMPLE 3 Using One Trigonometric Ratio to Find Them All

Let θ be an acute angle such that $\sin \theta = 5/6$. Evaluate the other five trigonometric functions of θ .



$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

FIGURE 4.11 How to create an acute angle θ such that $\sin \theta = 5/6$. (Example 3)

Common Calculator Errors When Evaluating Trig Functions

EXAMPLE 5 Solving a Right Triangle

A right triangle with a hypotenuse of 8 includes a 37° angle (Figure 4.17). Find the measures of the other two angles and the lengths of the other two sides.

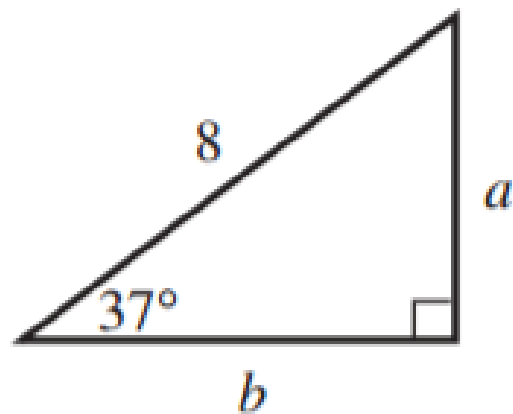


FIGURE 4.17 (Example 5)

EXAMPLE 6 Finding the Height of a Building

From a point 340 feet away from the base of the Peachtree Center Plaza in Atlanta, Georgia, the angle of elevation to the top of the building is 65° . (See Figure 4.18.) Find the height h of the building.

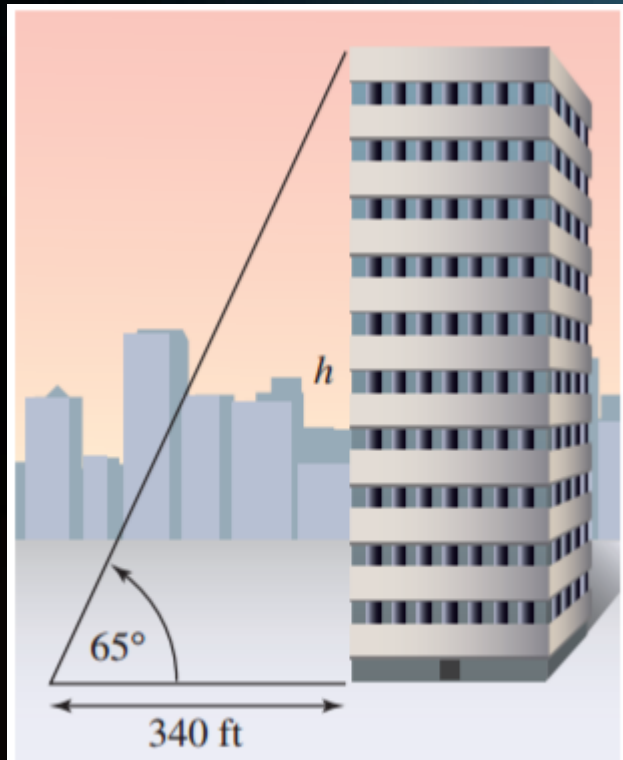


FIGURE 4.18 (Example 6)