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$$f(x) = a\sin(bx+c)+d$$

Save file as “sinusoid”

DEFINITION Sinusoid

A function is a **sinusoid** if it can be written in the form

$$f(x) = a \sin(bx + c) + d$$

where a, b, c, and d are constants and neither a nor b is 0.

DEFINITION Amplitude of a Sinusoid

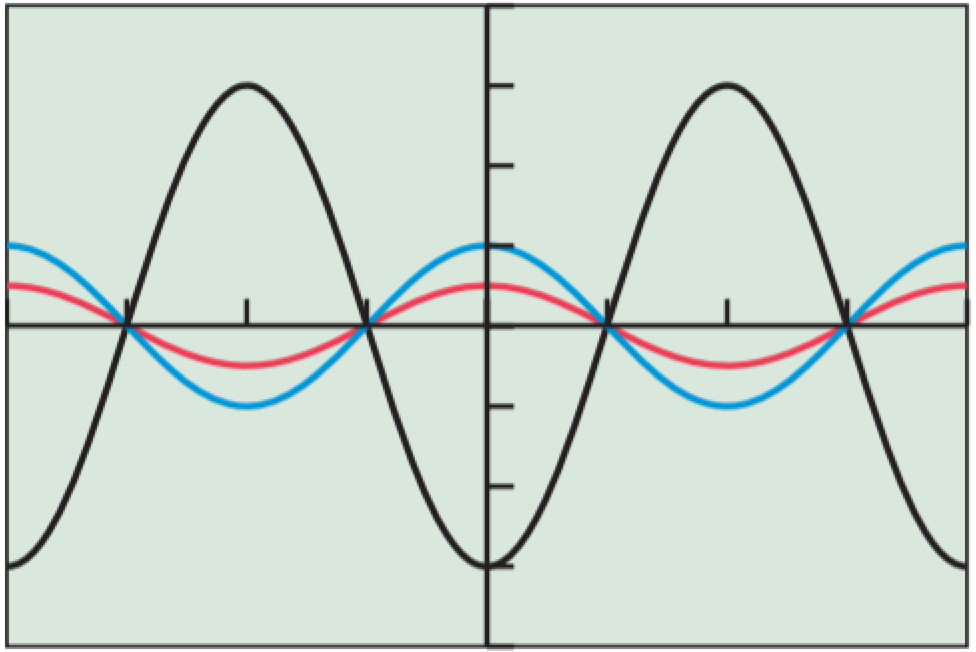
The **amplitude** of the sinusoid $f(x) = a \sin(bx + c) + d$ is $|a|$. Similarly, the amplitude of $f(x) = a \cos(bx + c) + d$ is $|a|$.

Graphically, the amplitude is half the height of the wave.

EXAMPLE 1 Vertical Stretch or Shrink and Amplitude

Find the amplitude of each function and use the language of transformations to describe how the graphs are related.

(a) $y_1 = \cos x$ **(b)** $y_2 = \frac{1}{2}\cos x$ **(c)** $y_3 = -3\cos x$



$[-2\pi, 2\pi]$ by $[-4, 4]$

Period of a Sinusoid

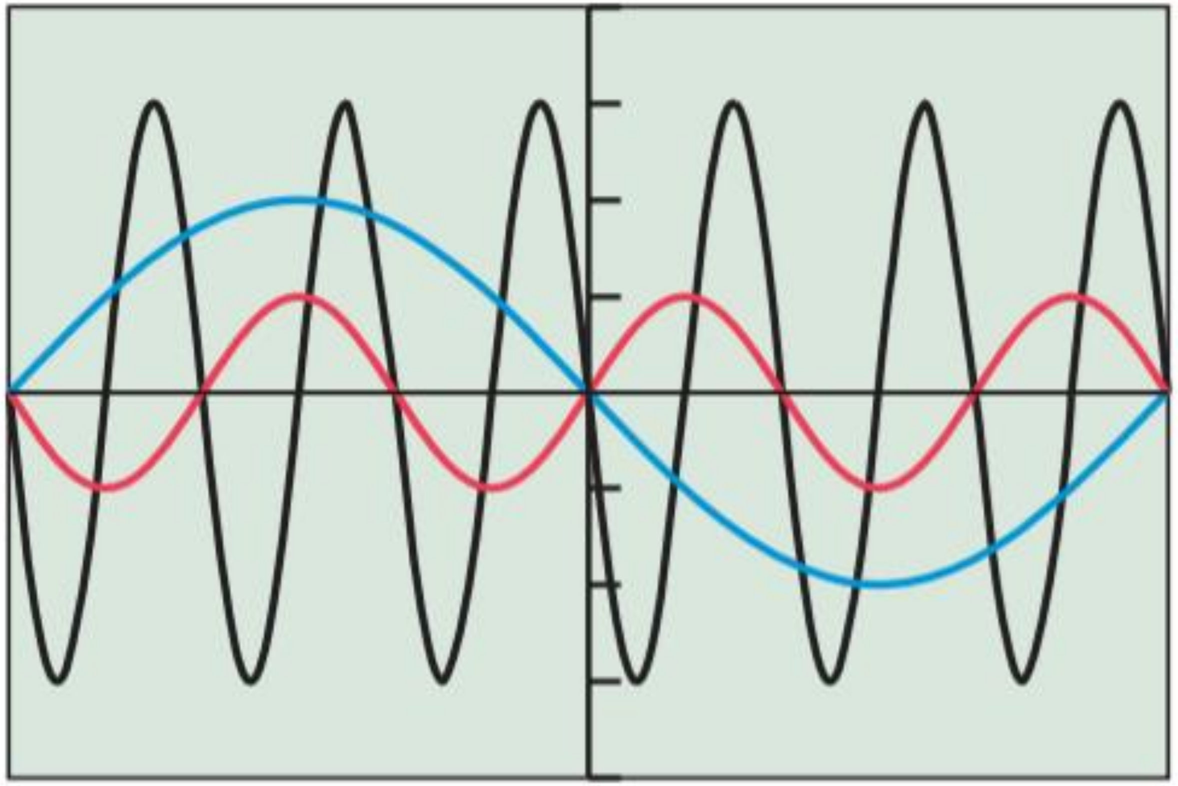
The period of the sinusoid $f(x) = a \sin(bx + c) + d$ is $2\pi/|b|$. Similarly, the period of $f(x) = a \cos(bx + c) + d$ is $2\pi/|b|$.

Graphically, the period is the length of one full cycle of the wave.

EXAMPLE 2 Horizontal Stretch or Shrink and Period

Find the period of each function and use the language of transformations to describe how the graphs are related.

(a) $y_1 = \sin x$ **(b)** $y_2 = -2 \sin\left(\dfrac{x}{3}\right)$ **(c)** $y_3 = 3 \sin(-2x)$



$[-3\pi, 3\pi]$ by $[-4, 4]$

FIGURE 4.40 Sinusoids (in this case, sine curves) of different amplitudes and periods. (Example 2)

In some applications, the *frequency* of a sinusoid is an important consideration. The frequency is simply the reciprocal of the period.

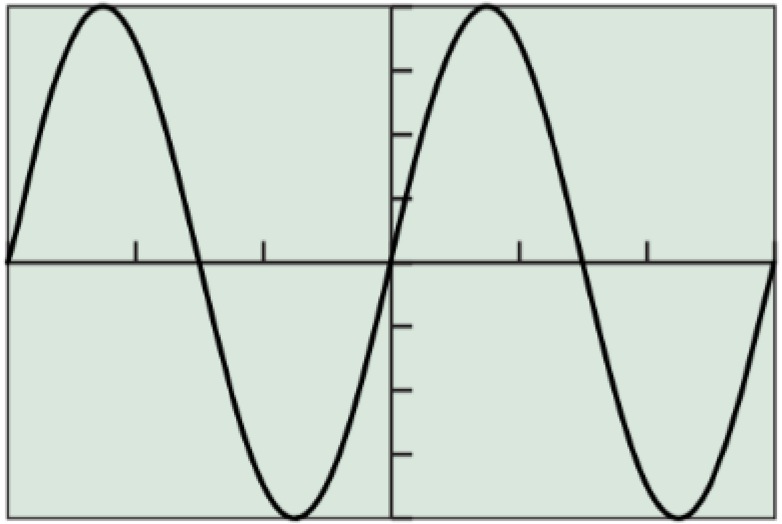
Frequency of a Sinusoid

The **frequency** of the sinusoid $f(x) = a \sin(bx + c) + d$ is $|b|/2\pi$. Similarly, the frequency of $f(x) = a \cos(bx + c) + d$ is $|b|/2\pi$.

Graphically, the frequency is the number of complete cycles the wave completes in a unit interval.

EXAMPLE 3 Finding the Frequency of a Sinusoid

Find the frequency of the function $f(x) = 4\sin(2x/3)$ and interpret its meaning graphically.



$[-3\pi, 3\pi]$ by $[-4, 4]$

SOLUTION The frequency is $(2/3) \div 2\pi = 1/(3\pi)$. This is the reciprocal of the period, which is 3π. The graphical interpretation is that the graph completes 1 full cycle per interval of length 3π. (That, of course, is what having a period of 3π is all about.) The graph is shown in Figure 4.41.

Recall from Section 1.5 that the graph of $y = f(x + c)$ is a translation of the graph of $y = f(x)$ by c units to the left when $c > 0$. That is exactly what happens with sinusoids, but using terminology with its roots in electrical engineering, we say that the wave undergoes a **phase shift** of $-c$.

EXAMPLE 4 Getting one Sinusoid from Another by a Phase Shift

(a) Write the cosine function as a phase shift of the sine function.

(b) Write the sine function as a phase shift of the cosine function.

EXAMPLE 5 Combining a Phase Shift with a Period Change

Construct a sinusoid with period $\pi/5$ and amplitude 6 that goes through $(2, 0)$.

$$y = 6 \sin(10(x - 2)) = 6 \sin(10x - 20).$$

Graphs of Sinusoids

The graphs of $y = a \sin(b(x - h)) + k$ and $y = a \cos(b(x - h)) + k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics:

$$\text{amplitude} = |a|;$$

$$\text{period} = \frac{2\pi}{|b|};$$

$$\text{frequency} = \frac{|b|}{2\pi}.$$

When compared to the graphs of $y = a \sin bx$ and $y = a \cos bx$, respectively, they also have the following characteristics:

a phase shift of h;

a vertical translation of k.

EXAMPLE 6 Constructing a Sinusoid by Transformations

Construct a sinusoid $y = f(x)$ that rises from a minimum value of $y = 5$ at $x = 0$ to a maximum value of $y = 25$ at $x = 32$. (See Figure 4.42.)