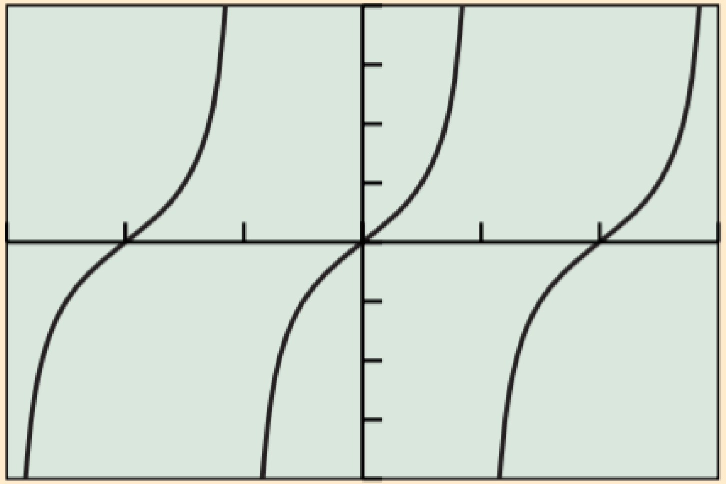
Graphs of Tangent, Cotangent, Secant, and Cosecant



$[-3\pi/2, 3\pi/2]$ by $[-4, 4]$

$f(x) = \tan x$

Domain: All reals except odd multiples of $\pi/2$

Range: All reals

Continuous (i.e., continuous on its domain)

Increasing on each interval in its domain

Symmetric with respect to the origin (odd).

Not bounded above or below

No local extrema

No horizontal asymptotes

Vertical asymptotes: $x = k \cdot (\pi/2)$ for all odd integers k

End behavior: $\lim_{x\to-\infty} \tan x$ and $\lim_{x\to\infty} \tan x$ do not exist. (The function values continually oscillate between $-\infty$ and ∞ and approach no limit.)

The Cotangent Function

The cotangent function is the reciprocal of the tangent function. Thus,

$$\cot x = \frac{\cos x}{\sin x}.$$

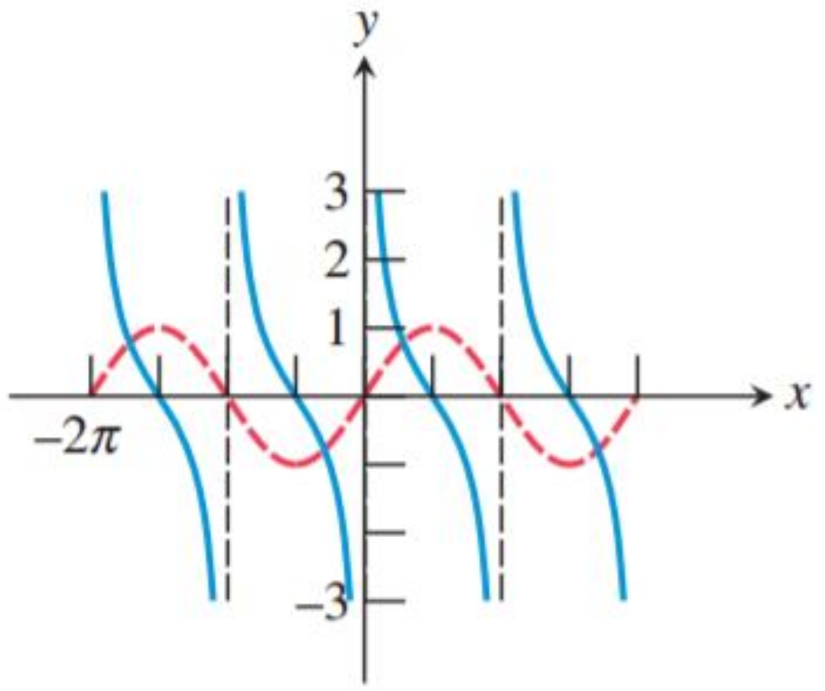


FIGURE 4.48 The cotangent has asymptotes at the zeros of the sine function.

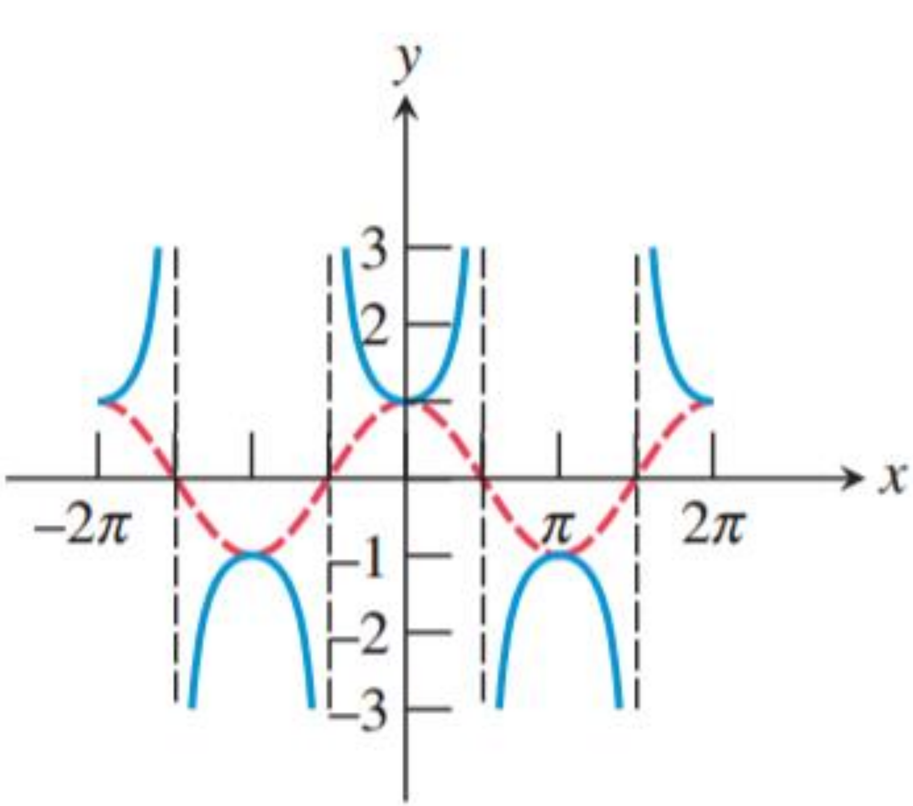
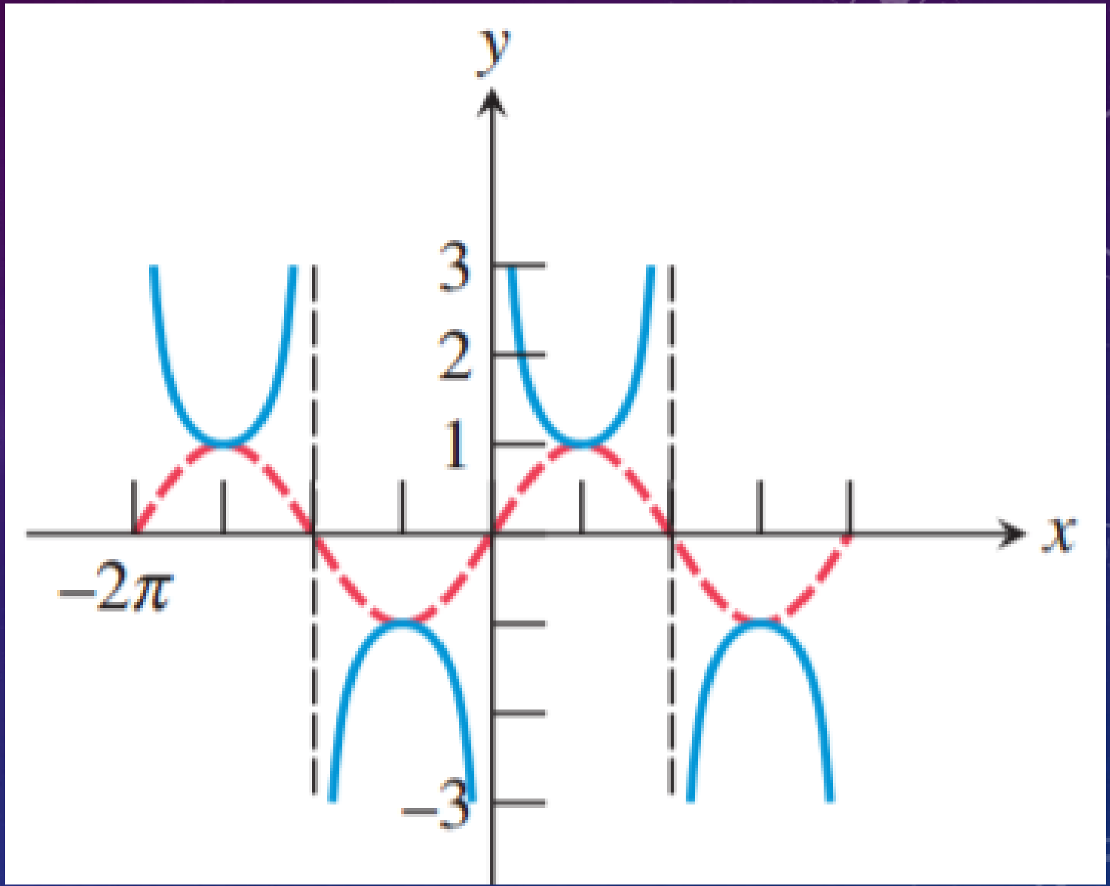


FIGURE 4.51 Characteristics of the secant function are inferred from the fact that it is the reciprocal of the cosine function.

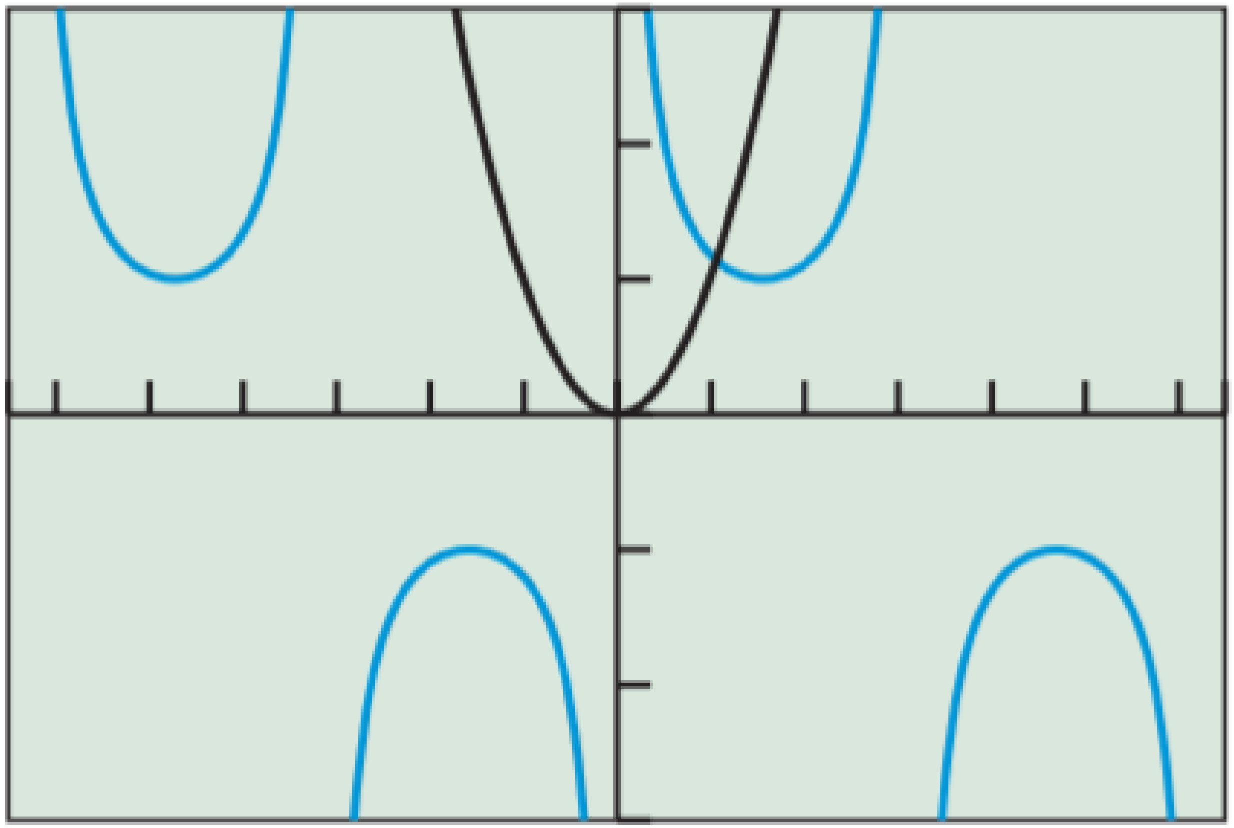


EXAMPLE 3 Solving a Trigonometric Equation Algebraically

Find the value of x between π and $3\pi/2$ that solves $\sec x = -2$.

EXAMPLE 4 Solving a Trigonometric Equation Graphically

Find the smallest positive number x such that $x^2 = \csc x$.



$[-6.5, 6.5]$ by $[-3, 3]$

Summary: Basic Trigonometric Functions

Function	Period	Domain	Range	Asymptotes	Zeros	Even/ Odd
$\sin x$	2π	All reals	$[-1, 1]$	None	$n\pi$	Odd
$\cos x$	2π	All reals	$[-1, 1]$	None	$\pi/2 + n\pi$	Even
$\tan x$	π	$x \neq \pi/2 + n\pi$	All reals	$x = \pi/2 + n\pi$	$n\pi$	Odd
$\cot x$	π	$x \neq n\pi$	All reals	$x = n\pi$	$\pi/2 + n\pi$	Odd
$\sec x$	2π	$x \neq \pi/2 + n\pi$	$(-\infty, -1] \cup [1, \infty)$	$x = \pi/2 + n\pi$	None	Even
$\csc x$	2π	$x \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$	$x = n\pi$	None	Odd