

EXAMPLE 1 Combining the Sine Function With x^2

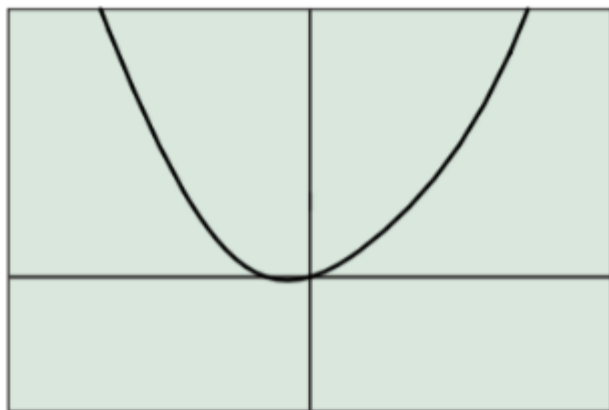
Graph each of the following functions for $-2\pi \leq x \leq 2\pi$, adjusting the vertical window as needed. Which of the functions appear to be periodic?

(a) $y = \sin x + x^2$

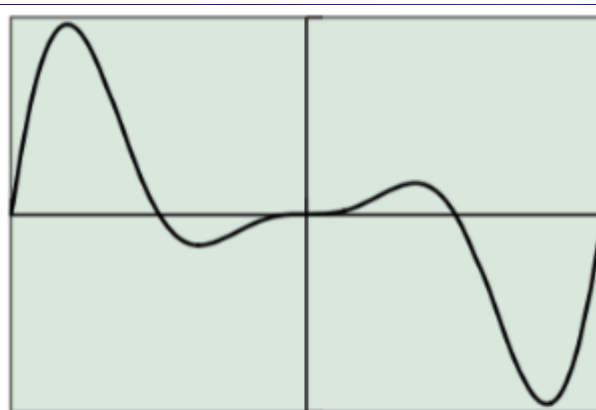
(b) $y = x^2 \sin x$

(c) $y = (\sin x)^2$

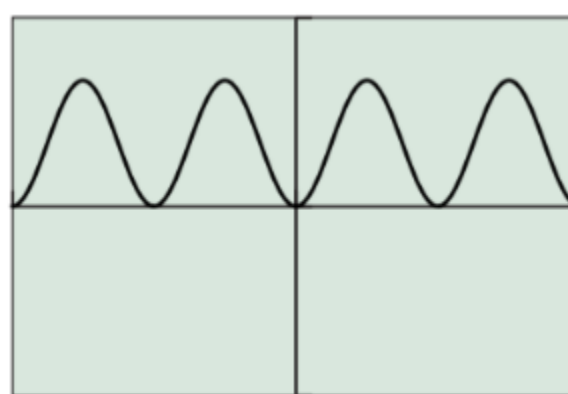
(d) $y = \sin(x^2)$



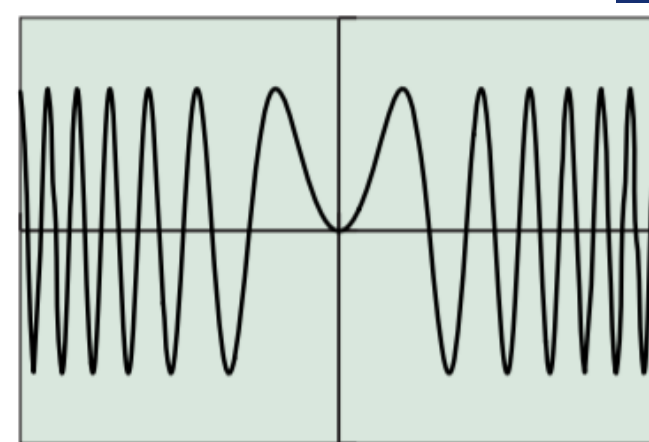
$[-2\pi, 2\pi]$ by $[-10, 20]$



$[-2\pi, 2\pi]$ by $[-25, 25]$



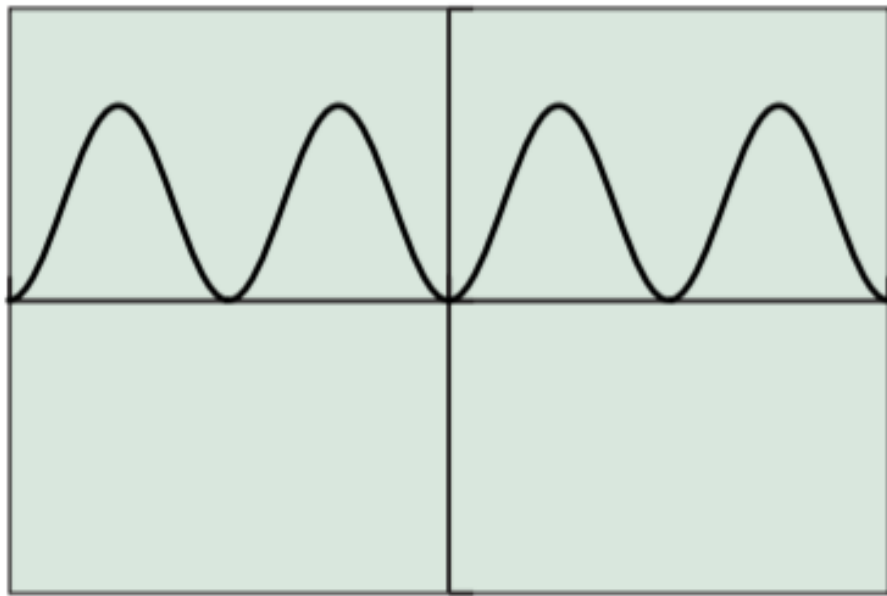
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$



EXAMPLE 2 Verifying Periodicity Algebraically

Verify algebraically that $f(x) = (\sin x)^2$ is periodic and determine its period graphically.

SOLUTION We use the fact that the period of the basic sine function is 2π , that is, $\sin(x + 2\pi) = \sin(x)$ for all x . It follows that

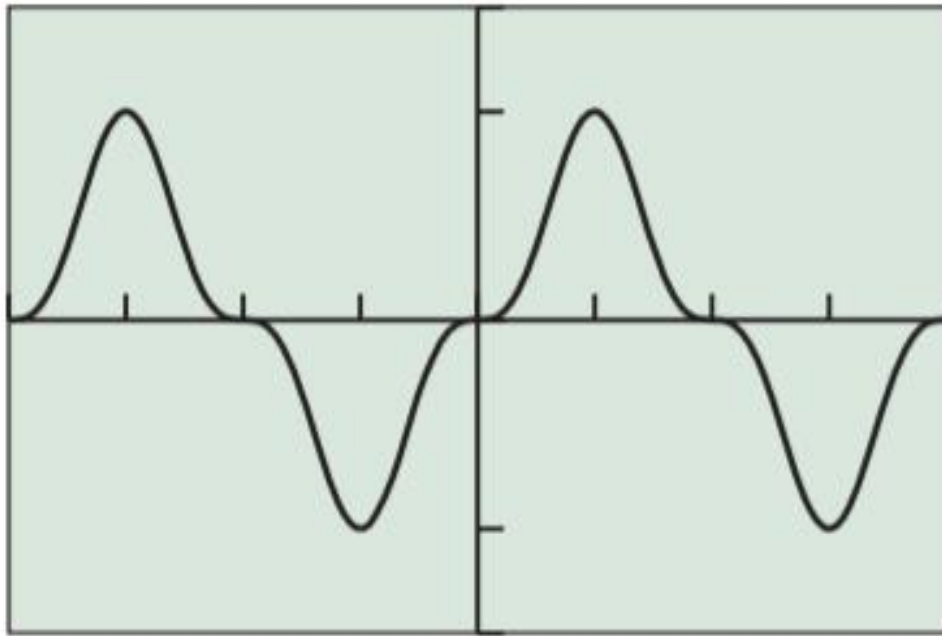


$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

$$\begin{aligned} f(x + 2\pi) &= (\sin(x + 2\pi))^2 \\ &= (\sin(x))^2 \quad \text{By periodicity of sine} \\ &= f(x) \end{aligned}$$

EXAMPLE 3 Composing $y = \sin x$ and $y = x^3$

Prove algebraically that $f(x) = \sin^3 x$ is periodic and find the period graphically. State the domain and range and sketch a graph showing two periods.



$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

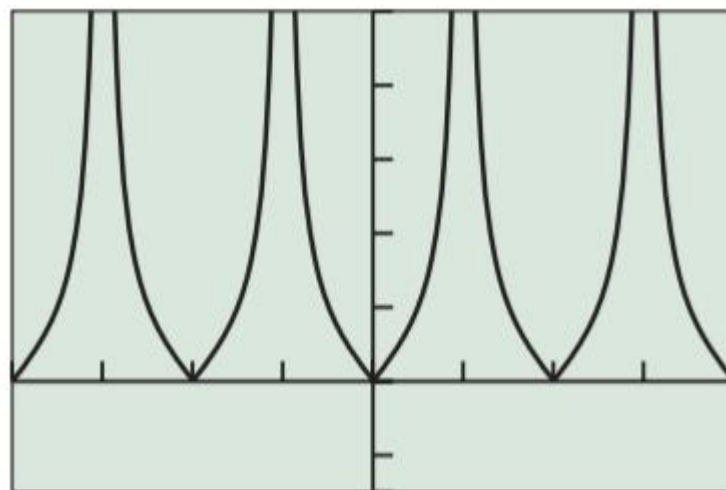
FIGURE 4.57 The graph of $f(x) = \sin^3 x$.
(Example 3)

EXAMPLE 4 Analyzing Nonnegative Periodic Functions

Find the domain, range, and period of each of the following functions. Sketch a graph showing four periods.

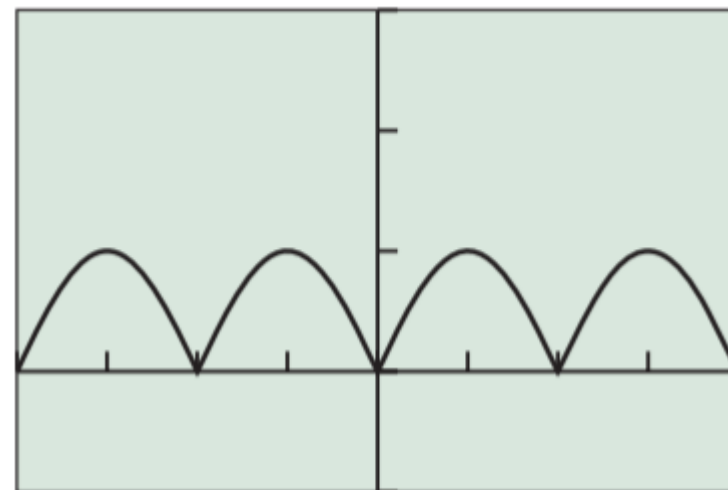
(a) $f(x) = |\tan x|$

(b) $g(x) = |\sin x|$



$[-2\pi, 2\pi]$ by $[-1.5, 5]$

FIGURE 4.59 $f(x) = |\tan x|$ has the same period as $y = \tan x$. (Example 4a)

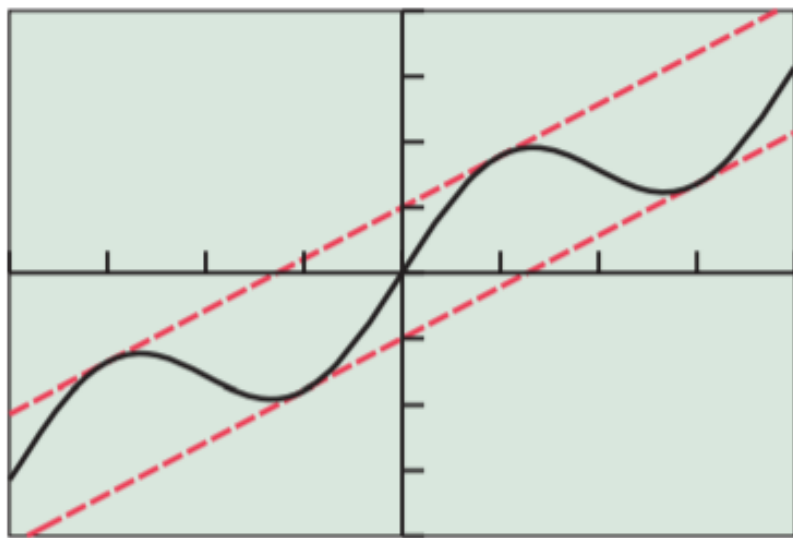


$[-2\pi, 2\pi]$ by $[-1, 3]$

FIGURE 4.60 $g(x) = |\sin x|$ has half the period of $y = \sin x$. (Example 4b)

EXAMPLE 5 Adding a Sinusoid to a Linear Function

The graph of $f(x) = 0.5x + \sin x$ oscillates between two parallel lines (Figure 4.61). What are the equations of the two lines?



$[-2\pi, 2\pi]$ by $[-4, 4]$

FIGURE 4.61 The graph of $f(x) = 0.5x + \sin x$ oscillates between the lines $y = 0.5x + 1$ and $y = 0.5x - 1$. Although the wave repeats its shape, it is not periodic. (Example 5)

EXAMPLE 6 Identifying a Sinusoid

Determine whether each of the following functions is or is not a sinusoid.

(a) $f(x) = 5 \cos x + 3 \sin x$

(b) $f(x) = \cos 5x + \sin 3x$

(c) $f(x) = 2 \cos 3x - 3 \cos 2x$

(d) $f(x) = a \cos\left(\frac{3x}{7}\right) - b \cos\left(\frac{3x}{7}\right) + c \sin\left(\frac{3x}{7}\right)$