EXAMPLE 7 Expressing the Sum of Sinusoids as a Sinusoid

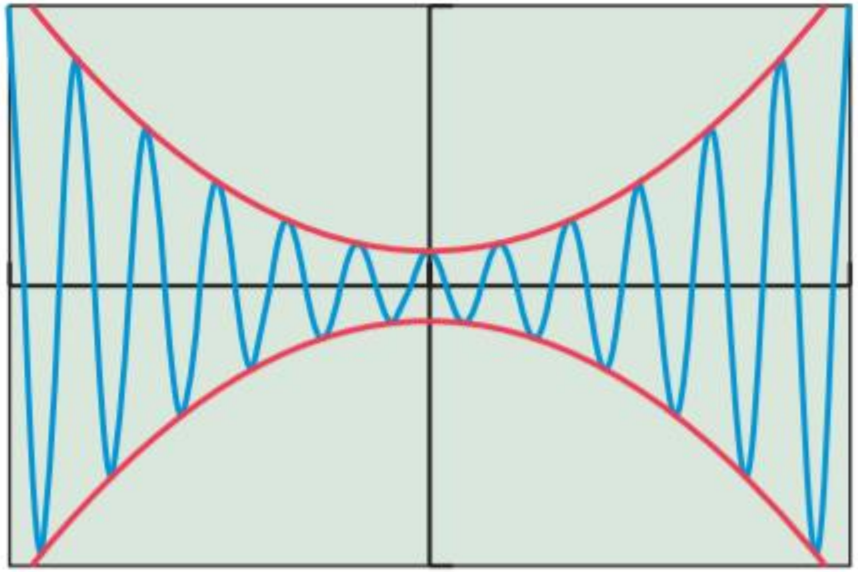
Let $f(x) = 2 \sin x + 5 \cos x$. From the discussion above, you should conclude that $f(x)$ is a sinusoid.

EXAMPLE 8 Showing a Function is Periodic but Not a Sinusoid

Show that $f(x) = \sin 2x + \cos 3x$ is periodic but not a sinusoid. Graph one period.

Because the values of sin *bt* and cos *bt* oscillate between −1 and 1, something interesting happens when either of these functions is multiplied by another function. For example, consider the function $y = (x^2 + 5)\cos 6x$, graphed in Figure 4.65. The (blue) graph of the function oscillates between the (red) graphs of $y = x^2 + 5$ and $y = -(x^2 + 5)$. The "squeezing" effect that can be seen near the origin is called **damping**.

Damped Oscillation



$[-2\pi, 2\pi]$ by $[-40, 40]$

FIGURE 4.65 The graph of $y = (x^2 + 5)\cos 6x$ shows **damped** oscillation.

Damped Oscillation

The graph of $y = f(x) \cos bx$ (or $y = f(x) \sin bx$) oscillates between the graphs of $y = f(x)$ and $y = -f(x)$. When this reduces the amplitude of the wave, it is called **damped oscillation**. The factor $f(x)$ is called the **damping factor**.

EXAMPLE 9 Identifying Damped Oscillation

For each of the following functions, determine if the graph shows damped oscillation. If it does, identify the damping factor and tell where the damping occurs.

(a) $f(x) = 2^{-x} \sin 4x$

(b) $f(x) = 3 \cos 2x$

(c) $f(x) = -2x \cos 2x$

EXAMPLE 10 A Damped Oscillating Spring

Dr. Sanchez's physics class collected data for an air table glider that oscillates between two springs. The class determined from the data that the equation

$$y = 0.22e^{-0.065t} \cos 2.4t$$

modeled the displacement y of the spring from its original position as a function of time t.

(a) Identify the damping factor and tell where the damping occurs.

(b) Approximately how long does it take for the spring to be damped so that $-0.1 \leq y \leq 0.1$?