Statements like "tan $\theta = \sin\theta/\cos\theta$" and "csc $\theta = 1/\sin\theta$" are trigonometric **identities** because they are true for all values of the variable for which both sides of the equation are defined. The set of all such values is called the **domain of validity** of the identity. We will spend much of this chapter exploring trigonometric identities, their proofs, their implications, and their applications.

Basic Trigonometric Identities

Basic Trigonometric Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

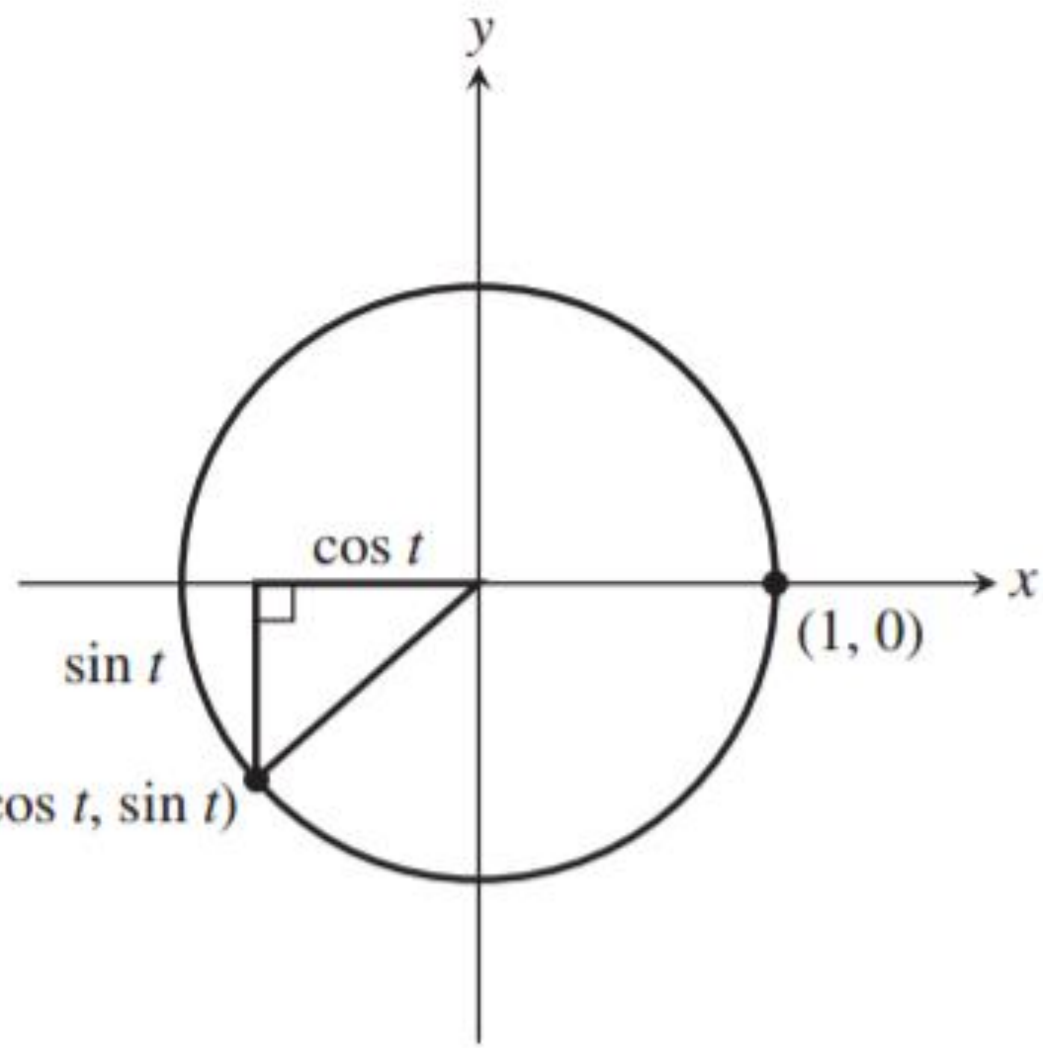

FIGURE 5.1 By the Pythagorean theorem, $(\cos t)^2 + (\sin t)^2 = 1$.

$$\frac{(\cos t)^2}{(\cos t)^2} + \frac{(\sin t)^2}{(\cos t)^2} = \frac{1}{(\cos t)^2}$$

$$1 + (\tan t)^2 = (\sec t)^2$$

$$\frac{(\cos t)^2}{(\sin t)^2} + \frac{(\sin t)^2}{(\sin t)^2} = \frac{1}{(\sin t)^2}$$

$$(\cot t)^2 + 1 = (\csc t)^2$$

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

EXAMPLE 1 Using Identities

Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = 5$ and $\cos\theta > 0$.

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta \qquad \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

Odd-Even Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

$$\csc(-x) = -\csc x \qquad \sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

EXAMPLE 2 Using More Identities

If $\cos\theta = 0.34$, find $\sin(\theta - \pi/2)$.

SOLUTION This problem can best be solved using identities.

$$\begin{aligned} \sin\left(\theta - \frac{\pi}{2}\right) &= -\sin\left(\frac{\pi}{2} - \theta\right) && \text{Sine is odd.} \\ &= -\cos\theta && \text{Cofunction identity} \\ &= -0.34 \end{aligned}$$

Simplifying Trigonometric Expressions

In calculus it is often necessary to deal with expressions that involve trigonometric functions. Some of those expressions start out looking fairly complicated, but it is often possible to use identities along with algebraic techniques (e.g., factoring or combining fractions over a common denominator) to *simplify* the expressions before dealing with them. In some cases the simplifications can be dramatic.

EXAMPLE 3 Simplifying by Factoring and Using Identities

Simplify the expression $\sin^3 x + \sin x \cos^2 x$.

EXAMPLE 4 Simplifying by Expanding and Using Identities

Simplify the expression $[(\sec x + 1)(\sec x - 1)]/\sin^2 x$.

EXAMPLE 5 Simplifying by Combining Fractions and Using Identities

Simplify the expression $\dfrac{\cos x}{1 - \sin x} - \dfrac{\sin x}{\cos x}$.

EXAMPLE 6 Solving a Trigonometric Equation

Find all values of x in the interval $[0, 2\pi)$ that solve $\cos^3 x / \sin x = \cot x$.

EXAMPLE 7 Solving a Trigonometric Equation by Factoring

Find all solutions to the trigonometric equation $2\sin^2 x + \sin x = 1$.