

In Exercises 52–57, match the function with an equivalent expression from the following list. Then confirm the match with a proof. (The matching is not one-to-one.)

(a)  $\sec^2 x \csc^2 x$

(b)  $\sec x + \tan x$

(c)  $2 \sec^2 x$

(d)  $\tan x \sin x$

(e)  $\sin x \cos x$

52. 
$$\frac{1 + \sin x}{\cos x}$$

54.  $\sec^2 x + \csc^2 x$

56. 
$$\frac{1}{\tan x + \cot x}$$

53.  $(1 + \sec x)(1 - \cos x)$

55. 
$$\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x}$$

57. 
$$\frac{1}{\sec x - \tan x}$$

$$22. \sin^2 \alpha - \cos^2 \alpha = 1 - 2 \cos^2 \alpha$$

$$23. \frac{1 + \tan^2 x}{\sin^2 x + \cos^2 x} = \sec^2 x$$

$$24. \frac{1}{\tan \beta} + \tan \beta = \sec \beta \csc \beta$$

$$25. \frac{\cos \beta}{1 + \sin \beta} = \frac{1 - \sin \beta}{\cos \beta}$$

$$26. \frac{\sec x + 1}{\tan x} = \frac{\sin x}{1 - \cos x}$$

$$27. \frac{\tan^2 x}{\sec x + 1} = \frac{1 - \cos x}{\cos x}$$

$$28. \frac{\cot v - 1}{\cot v + 1} = \frac{1 - \tan v}{1 + \tan v}$$

$$29. \cot^2 x - \cos^2 x = \cos^2 x \cot^2 x$$

$$30. \tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$$

$$31. \cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$$

$$32. \tan^4 t + \tan^2 t = \sec^4 t - \sec^2 t$$

**61. Multiple Choice** Which of these is an efficient first step in proving the identity  $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$ ?

(A)  $\frac{\sin x}{1 - \cos x} = \frac{\cos\left(\frac{\pi}{2} - x\right)}{1 - \cos x}$

(B)  $\frac{\sin x}{1 - \cos x} = \frac{\sin x}{\sin^2 x + \cos^2 x - \cos x}$

(C)  $\frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{\csc x}{\csc x}$

(D)  $\frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 - \cos x}{1 - \cos x}$

(E)  $\frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}$