Multiple-Angle Identities

Double-Angle Identities

Double-Angle Identities

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \begin{cases} \cos^2 u - \sin^2 u \\ 2 \cos^2 u - 1 \\ 1 - 2 \sin^2 u \end{cases}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

EXAMPLE 1   Proving a Double-Angle Identity

Prove the identity: $\sin 2u = 2 \sin u \cos u$.

Power-Reducing Identities

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

EXAMPLE 2 Proving an Identity

Prove the identity: $\cos^4 \theta - \sin^4 \theta = \cos 2\theta$.

EXAMPLE 3   Reducing a Power of 4

Rewrite $\cos^4 x$ in terms of trigonometric functions with no power greater than 1.

Half-Angle Identities

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \begin{cases} \pm\sqrt{\dfrac{1-\cos u}{1+\cos u}} \\[2ex] \dfrac{1-\cos u}{\sin u} \\[2ex] \dfrac{\sin u}{1+\cos u} \end{cases}$$

EXAMPLE 4   Using a Double-Angle Identity

Solve algebraically in the interval $[0, 2\pi)$: $\sin 2x = \cos x$.

EXAMPLE 5   Using Half-Angle Identities

Solve $\sin^2 x = 2 \sin^2 (x/2)$.

$$\cos x = 0 \quad \text{or} \quad \cos x = 1$$