The Law of Cosines

Law of Cosines

Let $\triangle ABC$ be any triangle with sides and angles labeled in the usual way (Figure 5.22).

Then

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

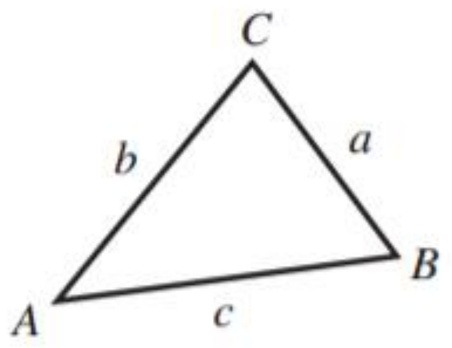


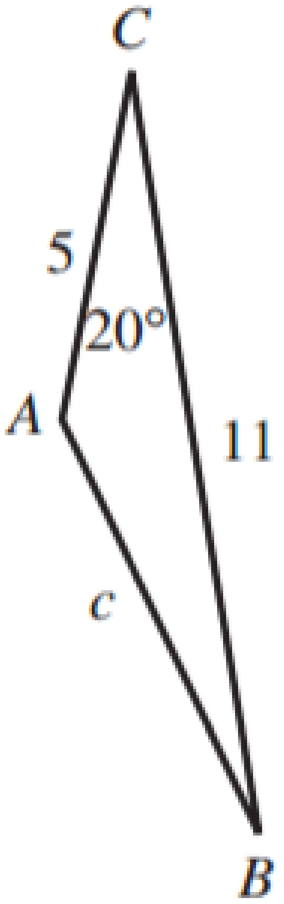
FIGURE 5.22 A triangle with the usual labeling (angles A, B, C; opposite sides a, b, c).

Solving Triangles (SAS, SSS)

While the Law of Sines is the tool we use to solve triangles in the AAS and ASA cases, the Law of Cosines is the required tool for SAS and SSS. (Both methods can be used in the SSA case, but remember that there might be 0, 1, or 2 triangles.)

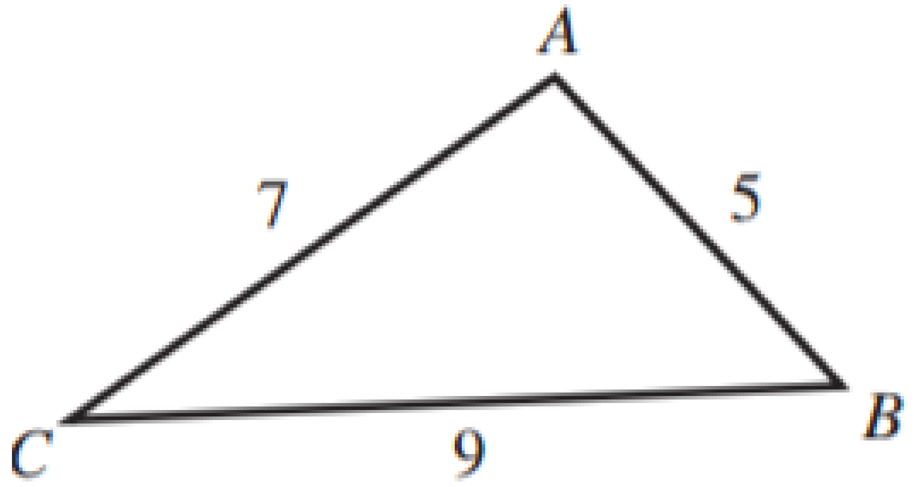
EXAMPLE 1 Solving a Triangle (SAS)

Solve $\triangle ABC$ given that $a = 11$, $b = 5$, and $C = 20°$.



EXAMPLE 2 Solving a Triangle (SSS)

Solve $\triangle ABC$ if $a = 9$, $b = 7$, and $c = 5$.



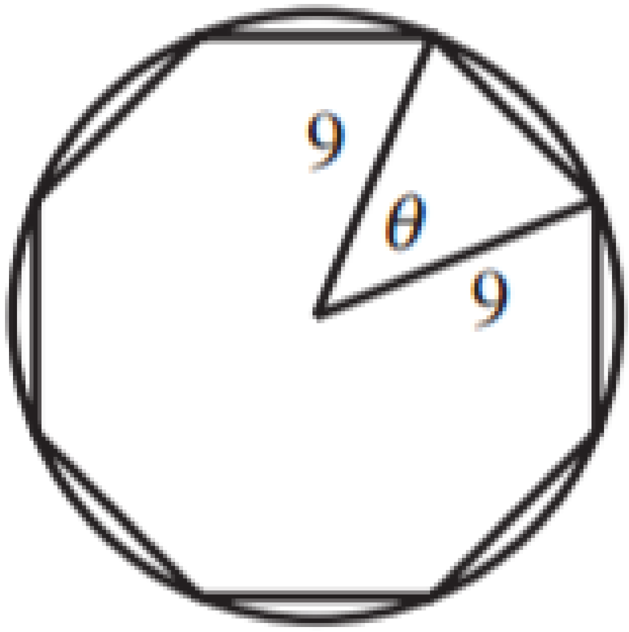
Triangle Area and Heron's Formula

Area of a Triangle

$$\triangle \text{ Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$

EXAMPLE 3 Finding the Area of a Regular Polygon

Find the area of a regular octagon (8 equal sides, 8 equal angles) inscribed inside a circle of radius 9 inches.



THEOREM Heron's Formula

Let a, b, and c be the sides of $\triangle ABC$, and let s denote the **semiperimeter**

$$(a + b + c)/2.$$

Then the area of $\triangle ABC$ is given by Area $= \sqrt{s(s-a)(s-b)(s-c)}$.

EXAMPLE 4 Using Heron's Formula

Find the area of a triangle with sides 13, 15, 18.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$