While the pair (a, b) determines a point in the plane, it also determines a **directed line segment** (or **arrow**) with its tail at the origin and its head at (a, b) (Figure 6.1). The length of this arrow represents magnitude, while the direction in which it points represents direction. Because in this context the ordered pair (a, b) represents a mathematical object with both magnitude and direction, we call it the **position vector of (a, b)**, and denote it as $\langle a, b \rangle$ to distinguish it from the point (a, b).

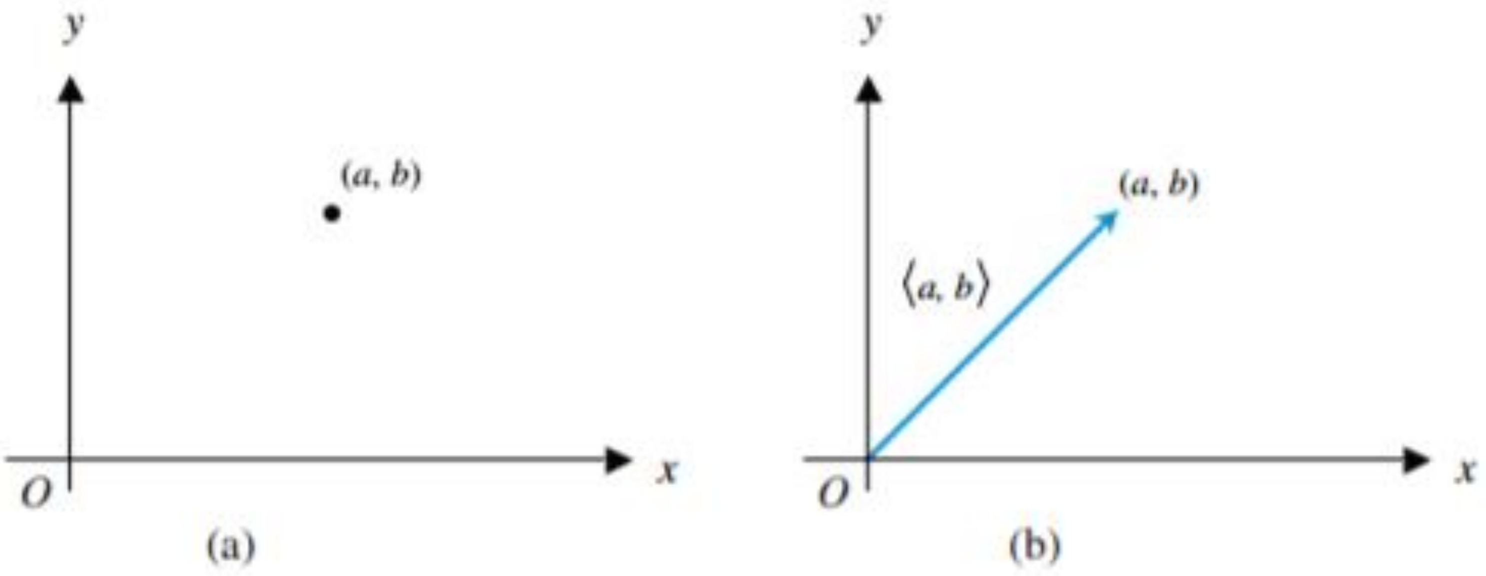


FIGURE 6.1 The point represents the ordered pair (a, b). The arrow (directed line segment) represents the vector $\langle a, b \rangle$.

DEFINITION Two-Dimensional Vector

A **two-dimensional vector v** is an ordered pair of real numbers, denoted in **component form** as $\langle a, b \rangle$. The numbers a and b are the **components** of the vector **v**. The **standard representation** of the vector $\langle a, b \rangle$ is the arrow from the origin to the point (a, b). The **magnitude** of **v** is the length of the arrow, and the **direction** of **v** is the direction in which the arrow is pointing. The vector $\mathbf{0} = \langle 0, 0 \rangle$, called the **zero vector**, has zero length and no direction.

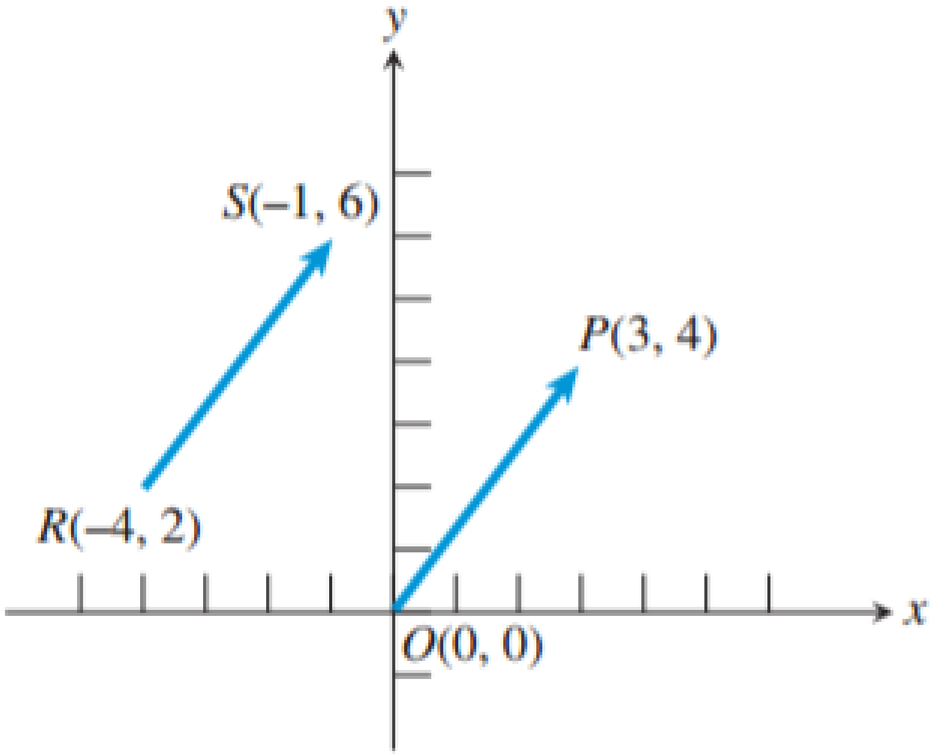


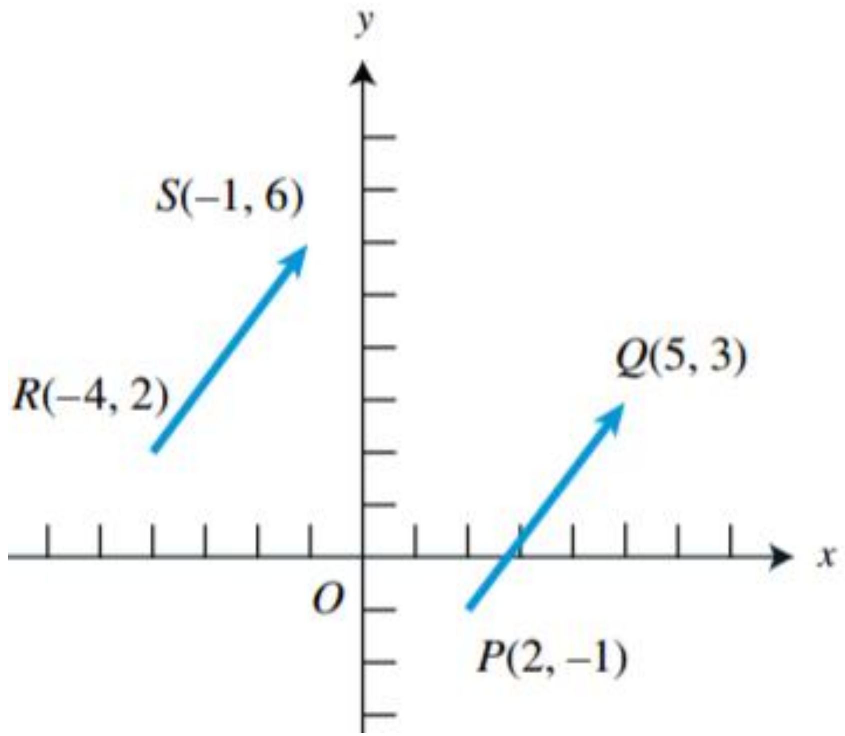
FIGURE 6.2 The arrows $\overrightarrow{RS}$ and $\overrightarrow{OP}$ both represent the vector $\langle 3, 4 \rangle$, as would any arrow with the same length pointing in the same direction. Such arrows are called *equivalent*.

Head Minus Tail (HMT) Rule

If an arrow has initial point (x_1, y_1) and terminal point (x_2, y_2), it represents the vector $\langle x_2 - x_1, y_2 - y_1 \rangle$.

EXAMPLE 1 Showing Arrows are Equivalent

Show that the arrow from $R = (-4, 2)$ to $S = (-1, 6)$ is equivalent to the arrow from $P = (2, -1)$ to $Q = (5, 3)$ (Figure 6.3).



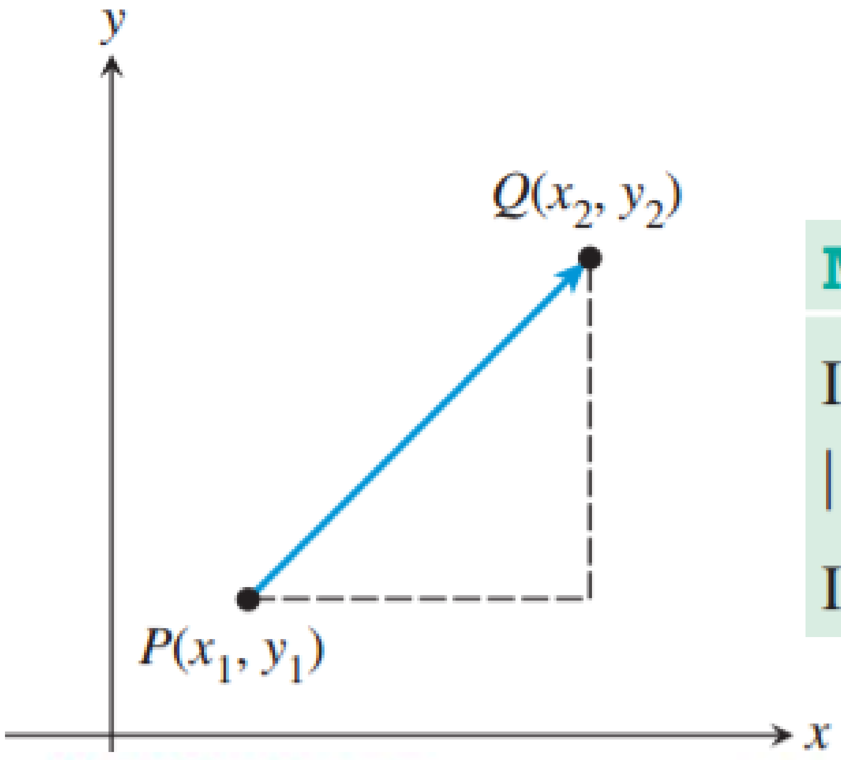


FIGURE 6.4 The magnitude of **v** is the length of the arrow $\overrightarrow{PQ}$, which is found using the distance formula: $|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Magnitude

If **v** is represented by the arrow from (x_1, y_1) to (x_2, y_2), then $|\mathbf{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

If $\mathbf{v} = \langle a, b \rangle$, then $|\mathbf{v}| = \sqrt{a^2 + b^2}$.

EXAMPLE 2 Finding Magnitude of a Vector

Find the magnitude of the vector **v** represented by $\overrightarrow{PQ}$, where $P = (-3, 4)$ and $Q = (-5, 2)$.

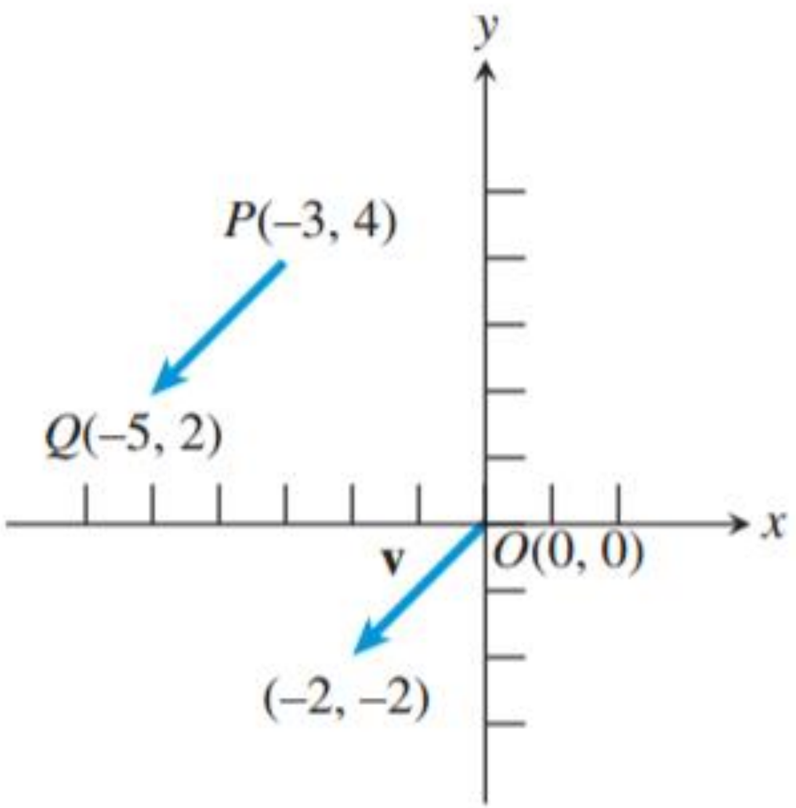


FIGURE 6.5 The vector **v** of Example 2.

DEFINITION **Vector Addition and Scalar Multiplication**

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a real number (scalar). The **sum** (or **resultant**) **of the vectors u and v** is the vector

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle.$$

The **product of the scalar k and the vector u** is

$$k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle.$$

In the **tail-to-head** representation, the standard representation of **u** points from the origin to (u_1, u_2). The arrow from (u_1, u_2) to $(u_1 + v_1, u_2 + v_2)$ represents **v** (as you can verify by the HMT Rule). The arrow from the origin to $(u_1 + v_1, u_2 + v_2)$ then represents **u** + **v** (Figure 6.6a).

In the **parallelogram** representation, the standard representations of **u** and **v** determine a parallelogram, the diagonal of which is the standard representation of **u** + **v** (Figure 6.6b).

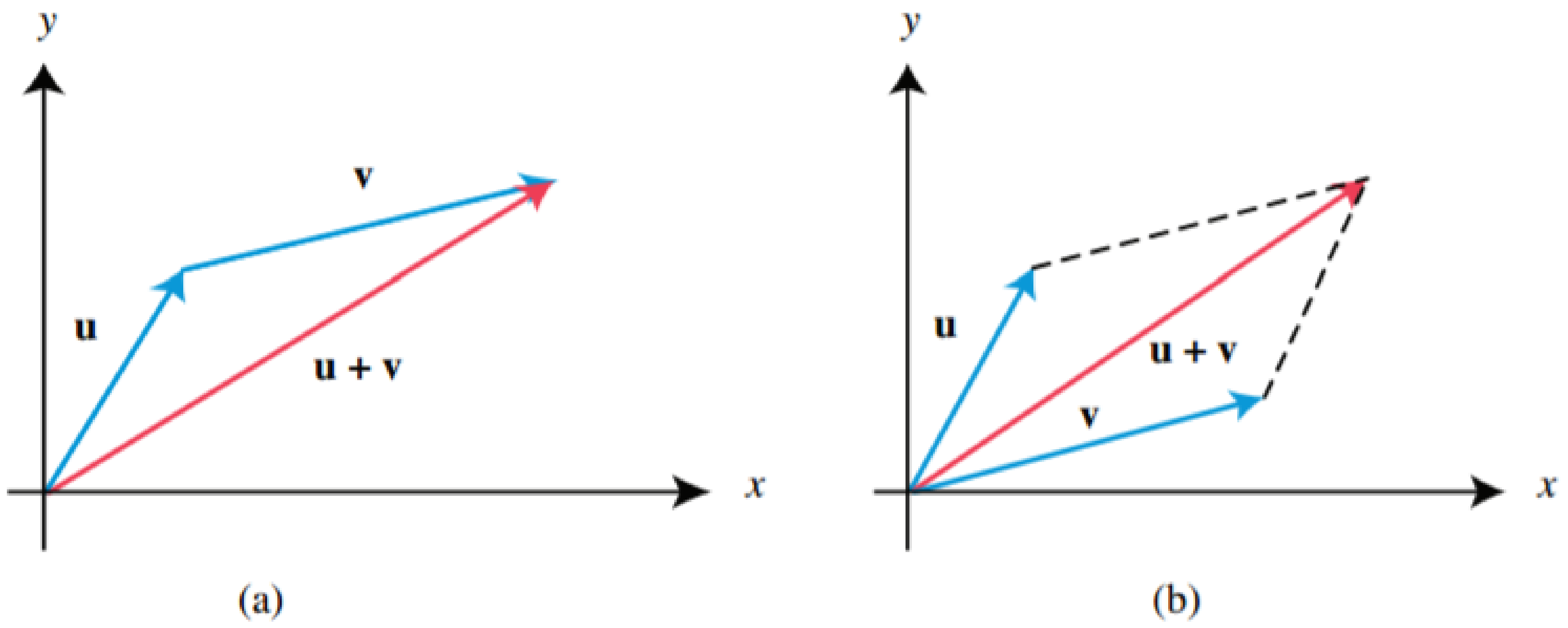


FIGURE 6.6 Two ways to represent vector addition geometrically: (a) tail-to-head, and (b) parallelogram.

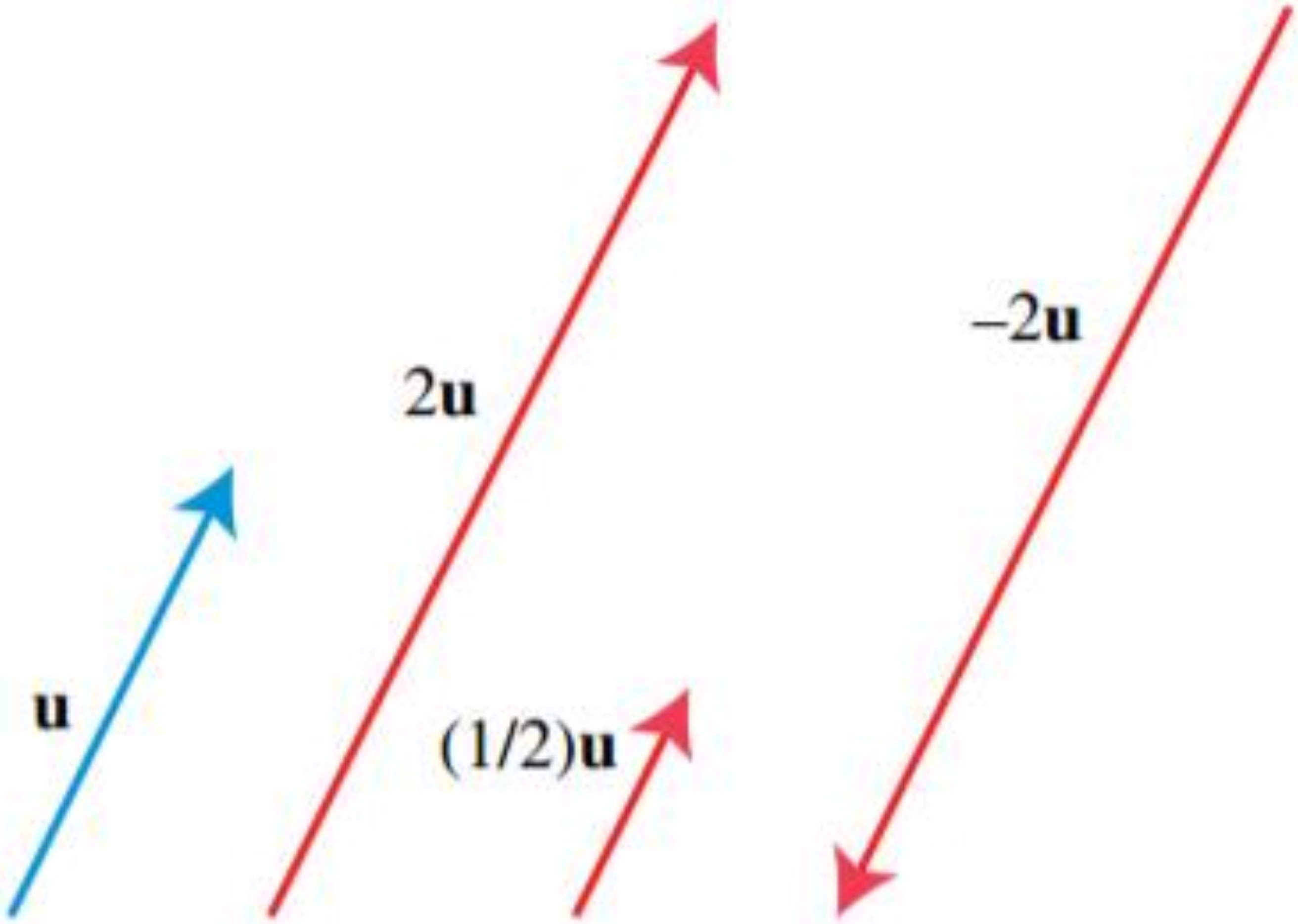


FIGURE 6.7 Representations of $\mathbf{u}$ and several scalar multiples of $\mathbf{u}$.

EXAMPLE 3 Performing Vector Operations

Let $\mathbf{u} = \langle -1, 3 \rangle$ and $\mathbf{v} = \langle 4, 7 \rangle$. Find the component form of the following vectors:

(a) $\mathbf{u} + \mathbf{v}$ **(b)** $3\mathbf{u}$ **(c)** $2\mathbf{u} + (-1)\mathbf{v}$

Unit Vectors

A vector $\mathbf{u}$ with length $|\mathbf{u}| = 1$ is a **unit vector**. If $\mathbf{v}$ is not the zero vector $\langle 0, 0 \rangle$, then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{|\mathbf{v}|}\mathbf{v}$$

is a **unit vector in the direction of v**. Unit vectors provide a way to represent the direction of any nonzero vector. Any vector in the direction of $\mathbf{v}$, or the opposite direction, is a scalar multiple of this unit vector $\mathbf{u}$.

EXAMPLE 4 Finding a Unit Vector

Find a unit vector in the direction of $\mathbf{v} = \langle -3, 2 \rangle$, and verify that it has length 1.