Dot Product of Vectors

DEFINITION Dot Product

The **dot product** or **inner product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2.$$

Properties of the Dot Product

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors and let c be a scalar.

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$
3. $\mathbf{0} \cdot \mathbf{u} = 0$
4. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

   $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$
5. $(c\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$

EXAMPLE 1 Finding Dot Products

Find each dot product.

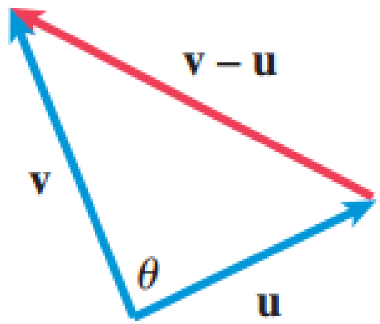
(a) $\langle 3, 4 \rangle \cdot \langle 5, 2 \rangle$

(b) $\langle 1, -2 \rangle \cdot \langle -4, 3 \rangle$

(c) $(2\mathbf{i} - \mathbf{j}) \cdot (3\mathbf{i} - 5\mathbf{j})$

EXAMPLE 2 Using Dot Product to Find Length

Use the dot product to find the length of the vector $\mathbf{u} = \langle 4, -3 \rangle$.



Angle Between Vectors

FIGURE 6.16 The angle θ between nonzero vectors $\mathbf{u}$ and $\mathbf{v}$.

THEOREM Angle Between Two Vectors

If θ is the angle between the nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, then

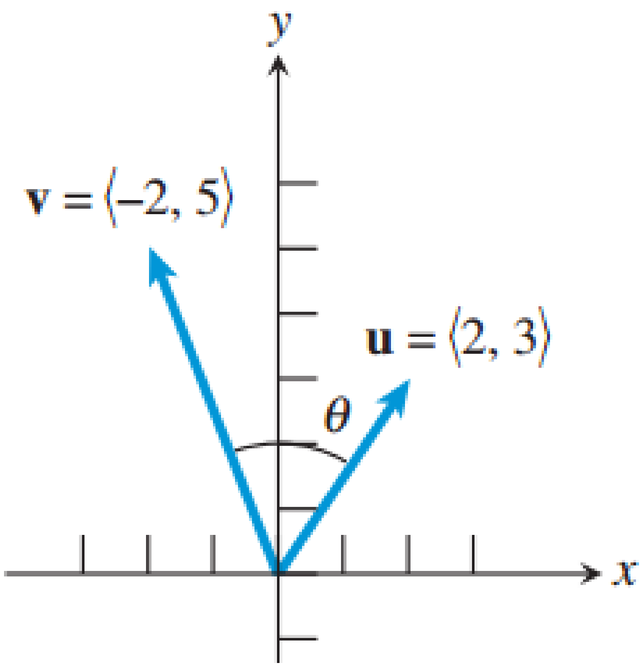
$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|}$$

$$\text{and } \theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}|\,|\mathbf{v}|}\right)$$

EXAMPLE 3 Finding the Angle Between Vectors

Find the angle between the vectors **u** and **v**.

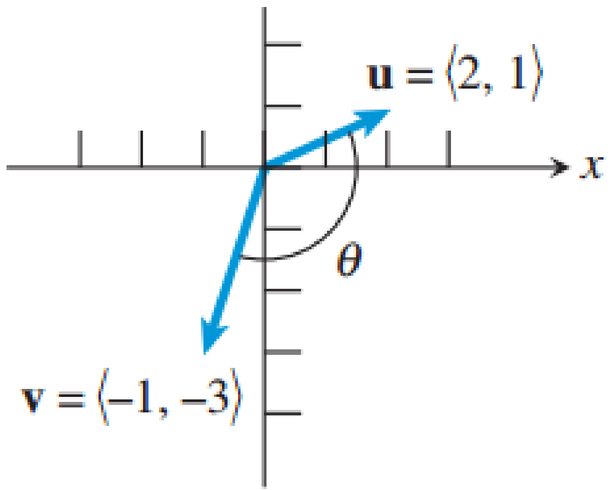
(a) $\mathbf{u} = \langle 2, 3 \rangle$, $\mathbf{v} = \langle -2, 5 \rangle$



EXAMPLE 3 Finding the Angle Between Vectors

Find the angle between the vectors **u** and **v**.

(b) $\mathbf{u} = \langle 2, 1 \rangle$, $\mathbf{v} = \langle -1, -3 \rangle$



DEFINITION Orthogonal Vectors

The vectors $\mathbf{u}$ and $\mathbf{v}$ are **orthogonal** if and only if $\mathbf{u} \cdot \mathbf{v} = 0$.

EXAMPLE 4 Proving Vectors are Orthogonal

Prove that the vectors $\mathbf{u} = \langle 2, 3 \rangle$ and $\mathbf{v} = \langle -6, 4 \rangle$ are orthogonal.