

DEFINITION Parametric Curve, Parametric Equations

The graph of the ordered pairs (x, y) where

$$x = f(t), \quad y = g(t)$$

are functions defined on an interval I of t -values is a **parametric curve**. The equations are **parametric equations** for the curve, the variable t is a **parameter**, and I is the **parameter interval**.

EXAMPLE 1 Graphing Parametric Equations

For the given parameter interval, graph the parametric equations

$$x = t^2 - 2, \quad y = 3t.$$

(a) $-3 \leq t \leq 1$

(b) $-2 \leq t \leq 3$

(c) $-3 \leq t \leq 3$

Eliminating the Parameter

When a curve is defined parametrically it is sometimes possible to *eliminate the parameter* and obtain a rectangular equation in x and y that represents the curve.

EXAMPLE 2 Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve

$$x = 1 - 2t, \quad y = 2 - t, \quad -\infty < t < \infty.$$

EXAMPLE 3 **Eliminating the Parameter**

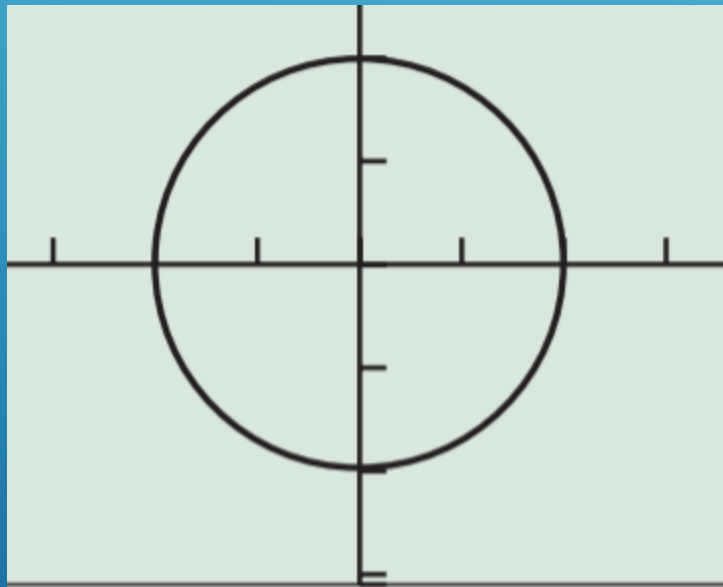
Eliminate the parameter and identify the graph of the parametric curve

$$x = t^2 - 2, \quad y = 3t.$$

EXAMPLE 4 Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve

$$x = 2 \cos t, \quad y = 2 \sin t, \quad 0 \leq t \leq 2\pi.$$



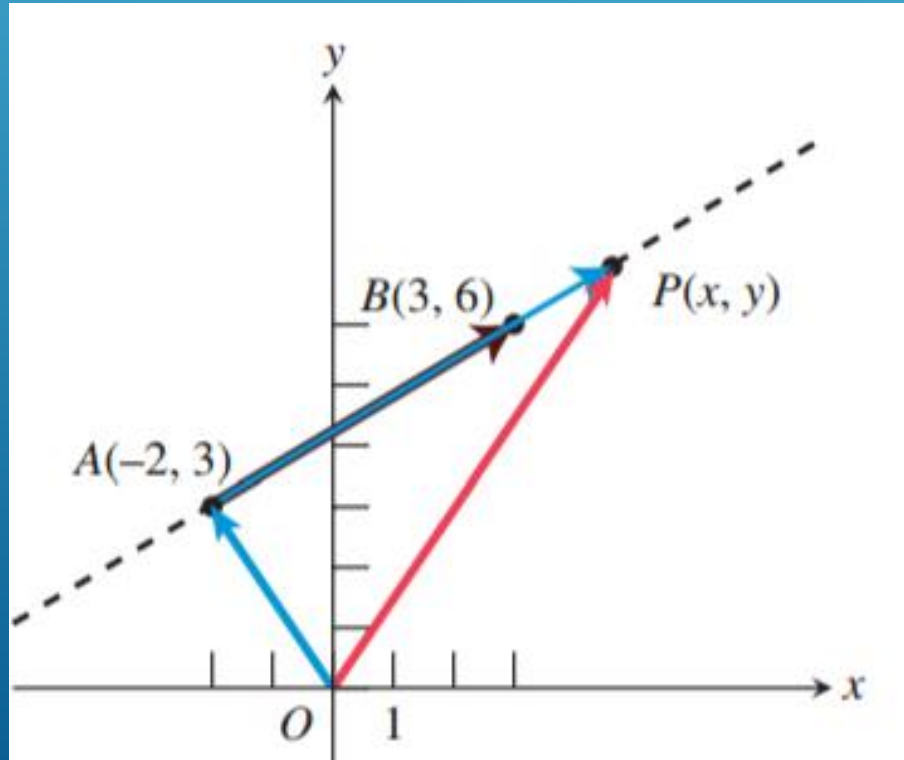
$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Lines and Line Segments

We can use vectors to help us find parametric equations for a line

EXAMPLE 5 Finding Parametric Equations for a Line

Find a parametrization of the line through the points $A = (-2, 3)$ and $B = (3, 6)$.



$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$

EXAMPLE 8 **Simulating Projectile Motion**

A distress flare is shot straight up from a ship's bridge 75 ft above the water with an initial velocity of 76 ft/sec. Graph the flare's height against time, give the height of the flare above water at each time, and simulate the flare's motion for each length of time.

(a) 1 sec

(b) 2 sec

(c) 4 sec

(d) 5 sec

Suppose that a baseball is thrown from a point y_0 feet above ground level with an initial speed of v_0 ft/sec at an angle θ with the horizontal (Figure 6.31). The initial velocity can be represented by the vector

$$\mathbf{v} = \langle v_0 \cos \theta, v_0 \sin \theta \rangle.$$

The path of the object is modeled by the parametric equations

$$x = (v_0 \cos \theta)t, \quad y = -16t^2 + (v_0 \sin \theta)t + y_0.$$

The x -component is simply

$$\text{distance} = (x\text{-component of initial velocity}) \times \text{time}.$$

The y -component is the familiar vertical projectile-motion equation using the y -component of initial velocity.

