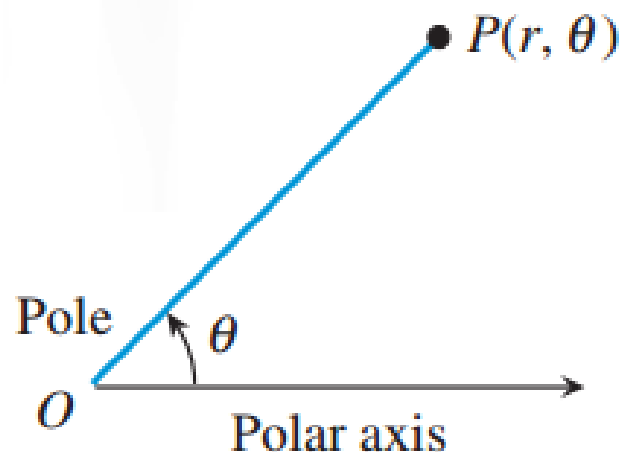


# Polar Coordinates

## Polar Coordinate System

A **polar coordinate system** is a plane with a point  $O$ , the **pole**, and a ray from  $O$ , the **polar axis**, as shown in Figure 6.35. Each point  $P$  in the plane is assigned as **polar coordinates** follows:  $r$  is the **directed distance** from  $O$  to  $P$ , and  $\theta$  is the **directed angle** whose initial side is on the polar axis and whose terminal side is on the line  $OP$ .

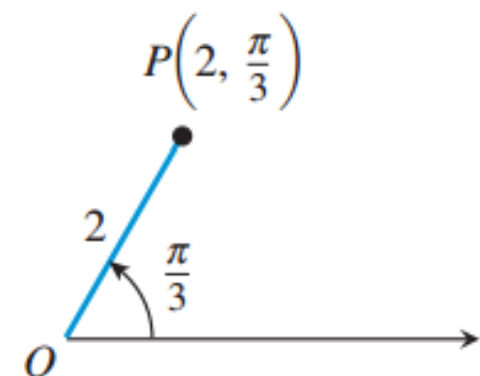
As in trigonometry, we measure  $\theta$  as positive when moving counterclockwise and negative when moving clockwise. If  $r > 0$ , then  $P$  is on the terminal side of  $\theta$ . If  $r < 0$ , then  $P$  is on the terminal side of  $\theta + \pi$ . We can use radian or degree measure for the angle  $\theta$  as illustrated in Example 1.



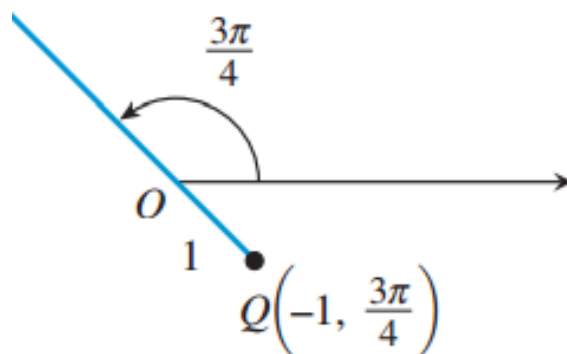
## EXAMPLE 1 Plotting Points in the Polar Coordinate System

Plot the points with the given polar coordinates.

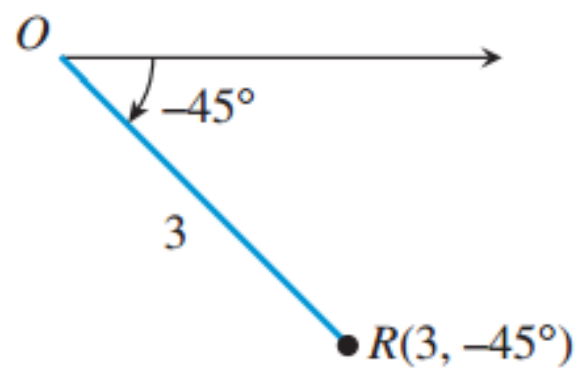
(a)  $P(2, \pi/3)$



(b)  $Q(-1, 3\pi/4)$

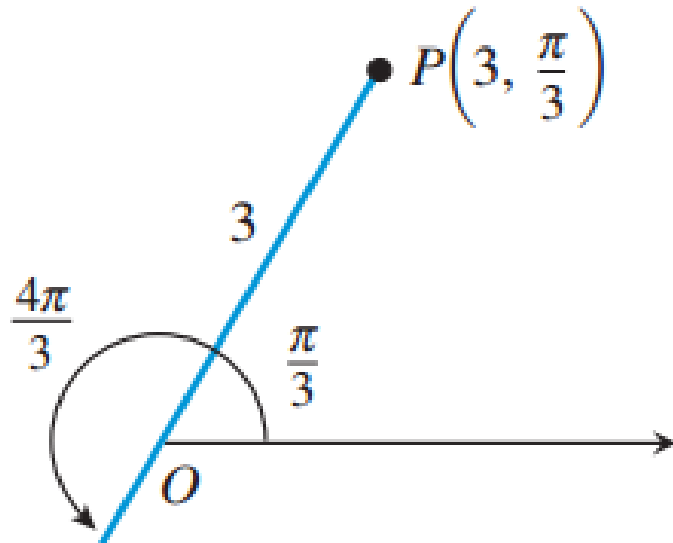


(c)  $R(3, -45^\circ)$



## EXAMPLE 2 Finding all Polar Coordinates for a Point

If the point  $P$  has polar coordinates  $(3, \pi/3)$ , find all polar coordinates for  $P$ .

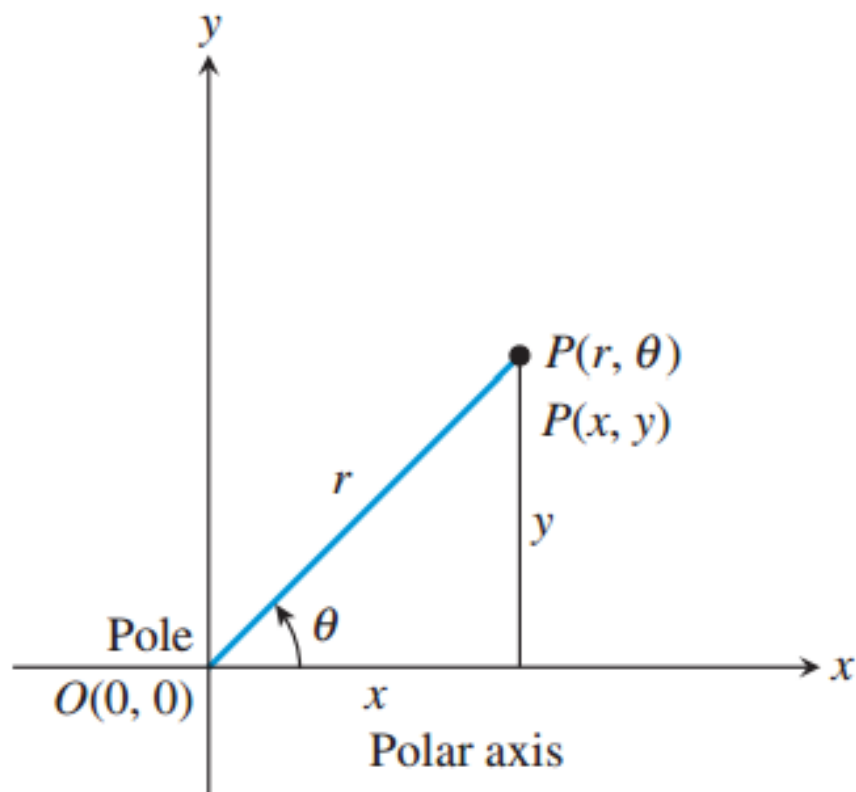


## Coordinate Conversion

Let the point  $P$  have polar coordinates  $(r, \theta)$  and rectangular coordinates  $(x, y)$ . Then

$$x = r \cos \theta, \quad r^2 = x^2 + y^2,$$

$$y = r \sin \theta, \quad \tan \theta = \frac{y}{x}.$$



### **EXAMPLE 3**    **Converting from Polar to Rectangular Coordinates**

Find the rectangular coordinates of the points with the given polar coordinates.

(a)  $P(3, 5\pi/6)$

(b)  $Q(2, -200^\circ)$

## EXAMPLE 4 Converting from Rectangular to Polar Coordinates

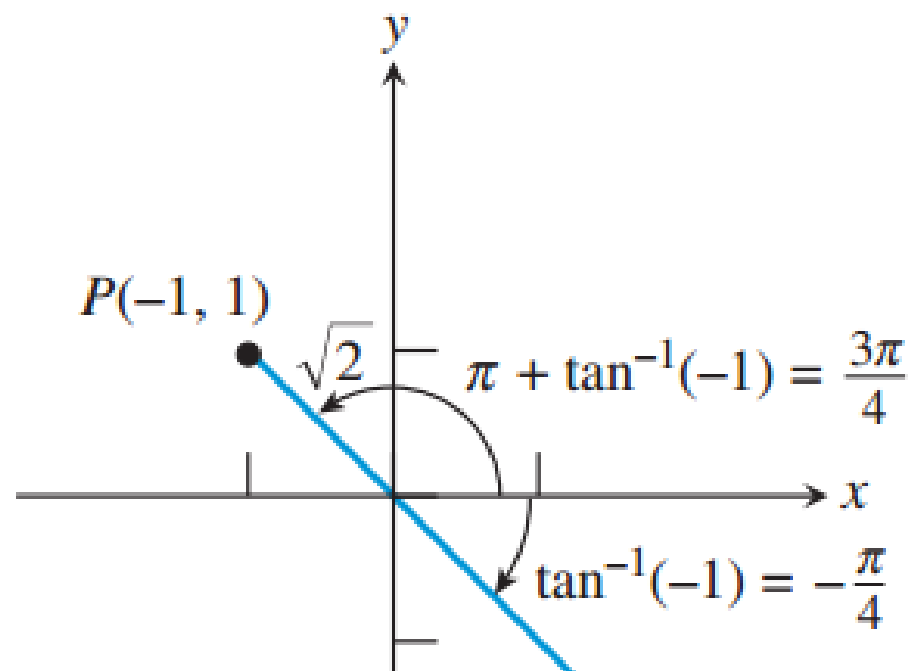
Find two polar coordinate pairs for the points with given rectangular coordinates.

(a)  $P(-1, 1)$

(b)  $Q(-3, 0)$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$



## Equation Conversion

We can use the Coordinate Conversion Equations to convert polar form to rectangular form and vice versa. For example, the polar equation  $r = 4 \cos \theta$  can be converted to rectangular form as follows:

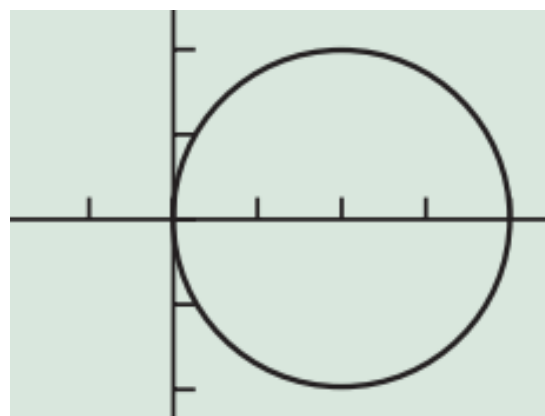
$$r = 4 \cos \theta$$

$$r^2 = 4r \cos \theta$$

$$x^2 + y^2 = 4x$$

$$x^2 - 4x + 4 + y^2 = 4$$

$$(x - 2)^2 + y^2 = 4$$



$$r^2 = x^2 + y^2, r \cos \theta = x$$

Subtract  $4x$  and add 4.

Factor.