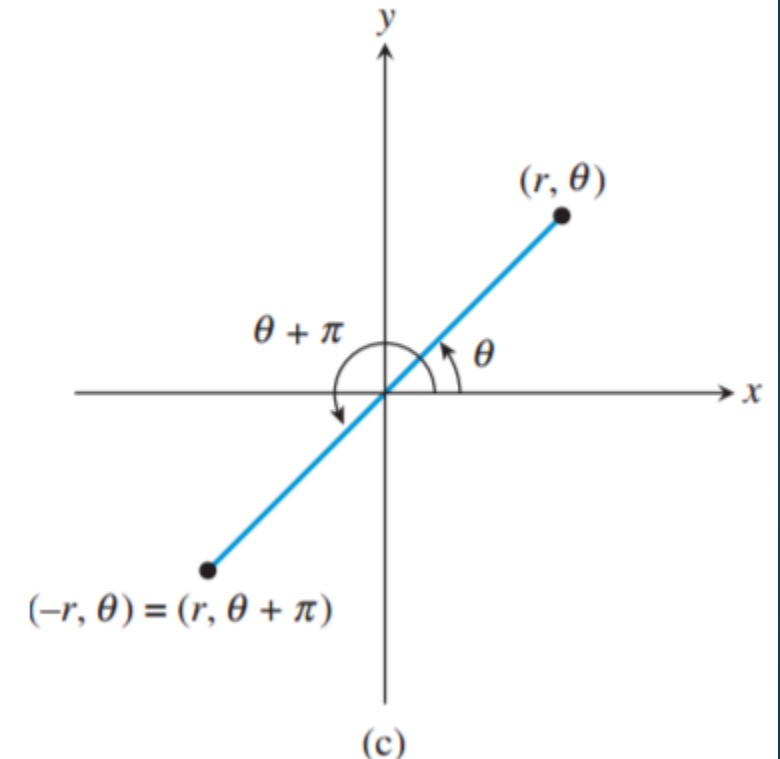
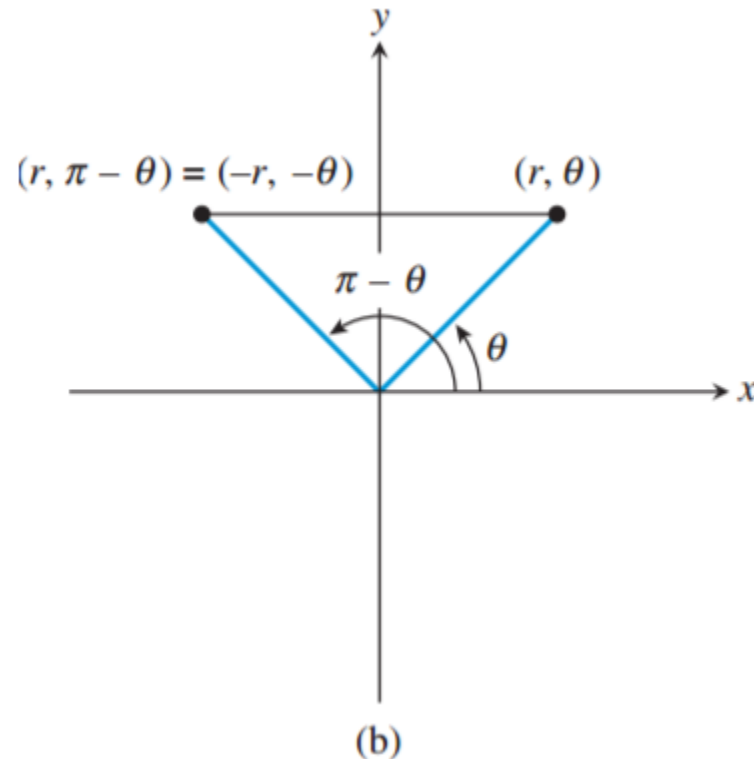
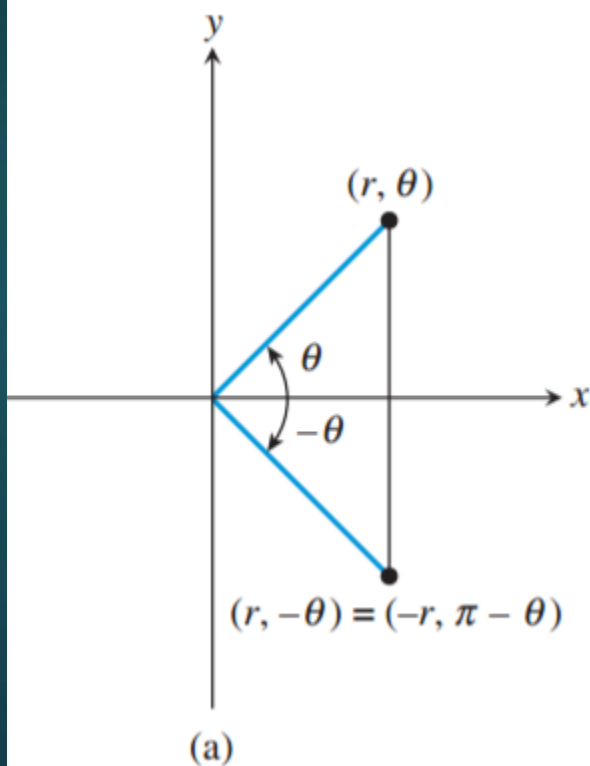


Graphs of Polar Equations

The three types of symmetry figures to be considered will have are:

1. The x -axis (polar axis) as a line of symmetry (Figure 6.45a).
2. The y -axis (the line $\theta = \pi/2$) as a line of symmetry (Figure 6.45b).
3. The origin (the pole) as a point of symmetry (Figure 6.45c).



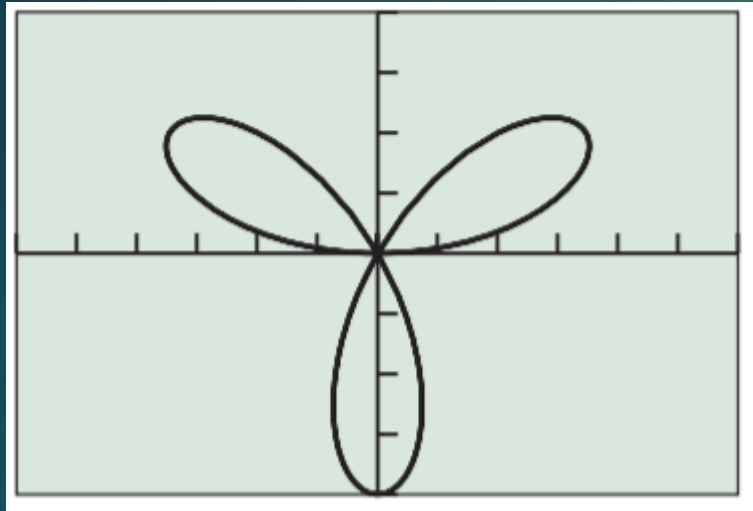
Symmetry Tests for Polar Graphs

The graph of a polar equation has the indicated symmetry if either replacement produces an equivalent polar equation.

To Test for Symmetry	Replace	By
1. about the x -axis,	(r, θ)	$(r, -\theta)$ or $(-r, \pi - \theta)$.
2. about the y -axis,	(r, θ)	$(-r, -\theta)$ or $(r, \pi - \theta)$.
3. about the origin,	(r, θ)	$(-r, \theta)$ or $(r, \theta + \pi)$.

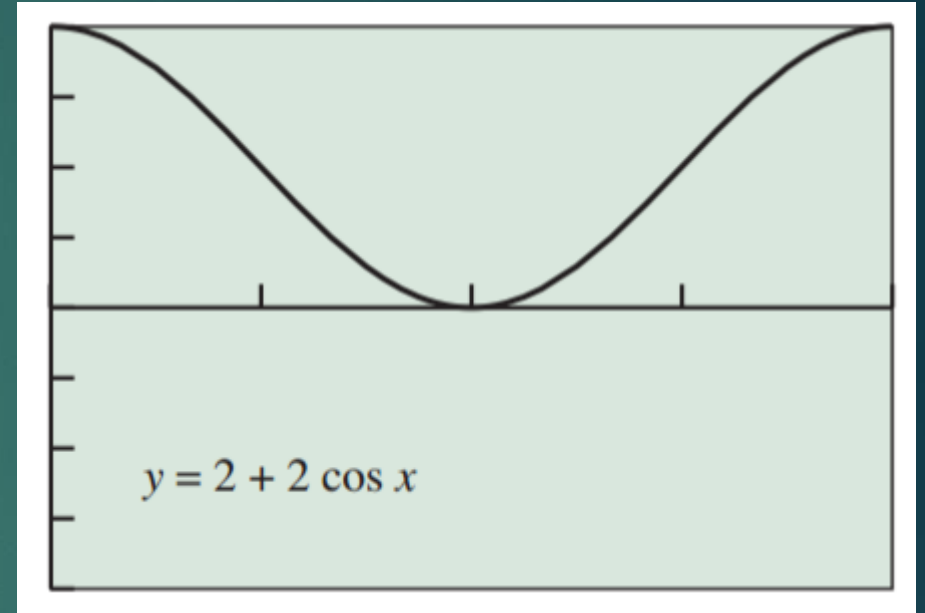
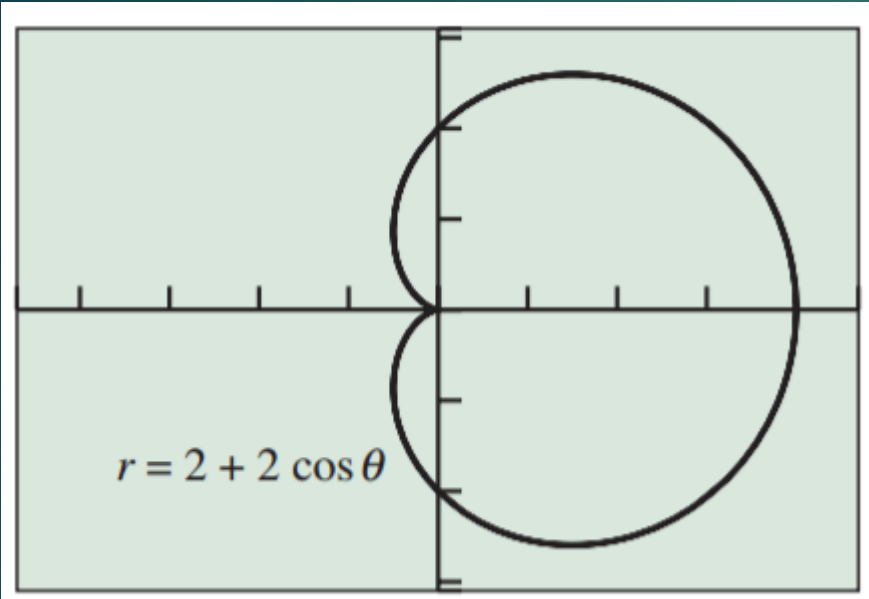
EXAMPLE 1 Testing for Symmetry

Use the symmetry tests to prove that the graph of $r = 4 \sin 3\theta$ is symmetric about the y -axis.



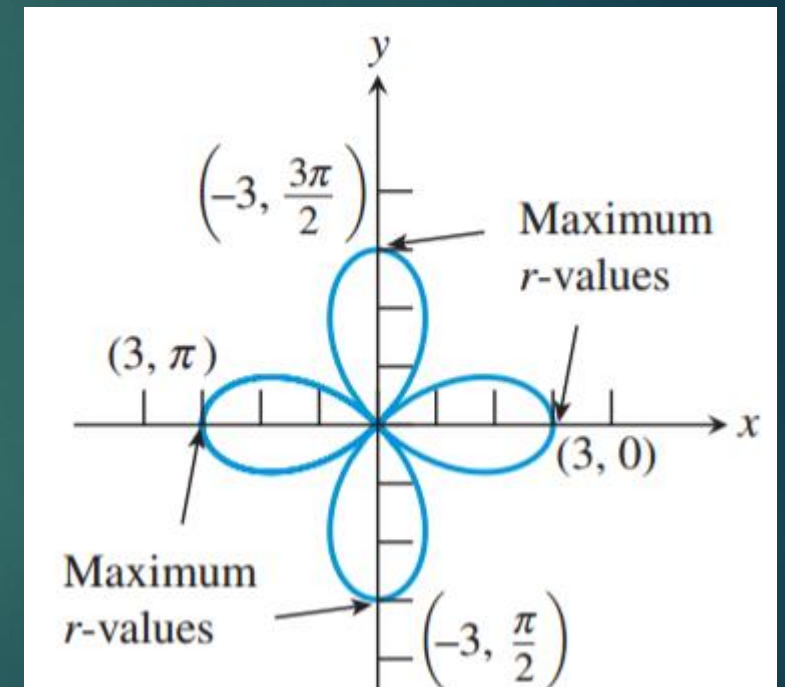
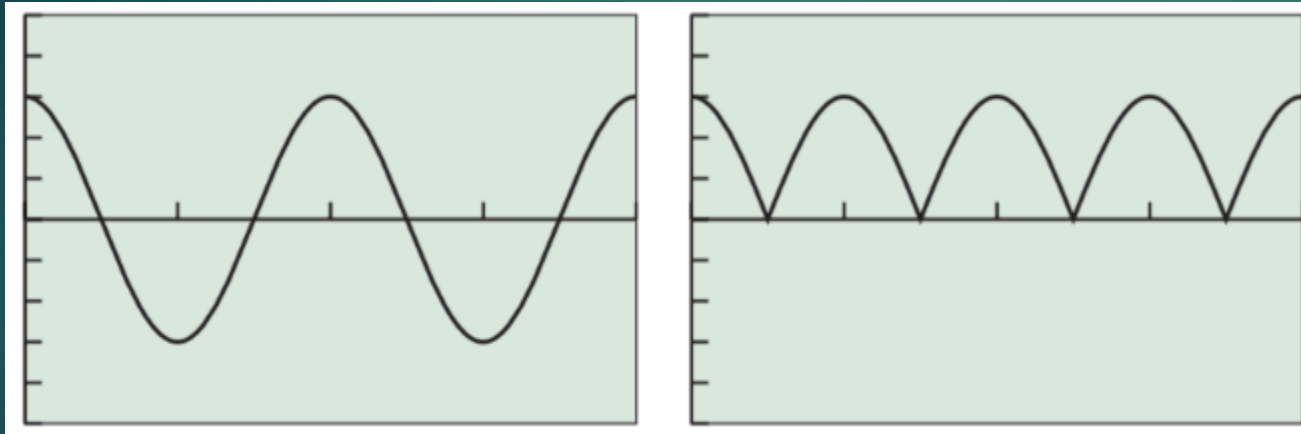
EXAMPLE 2 Finding Maximum r -Values

Find the maximum r -value of $r = 2 + 2 \cos \theta$.



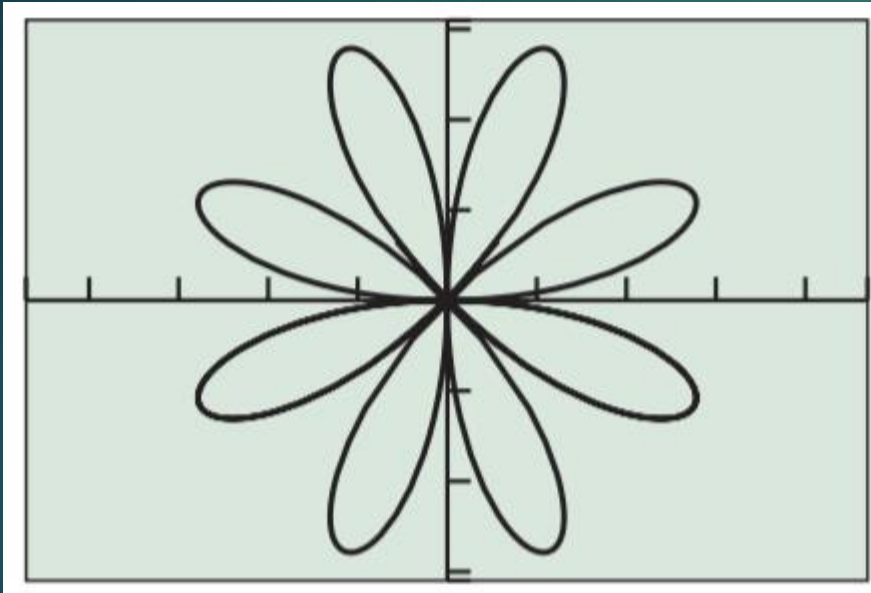
EXAMPLE 3 Finding Maximum r -Values

Identify the points on the graph of $r = 3 \cos 2\theta$ for $0 \leq \theta \leq 2\pi$ that give maximum r -values.



EXAMPLE 4 Analyzing a Rose Curve

Analyze the graph of the rose curve $r = 3 \sin 4\theta$.



Graphs of Rose Curves

The graphs of $r = a \cos n\theta$ and $r = a \sin n\theta$, where $n > 1$ is an integer, have the following characteristics:

Domain: All reals

Range: $[-|a|, |a|]$

Continuous

Symmetry: n even, symmetric about x -, y -axis, origin
 n odd, $r = a \cos n\theta$ symmetric about x -axis
 n odd, $r = a \sin n\theta$ symmetric about y -axis

Bounded

Maximum r -value: $|a|$

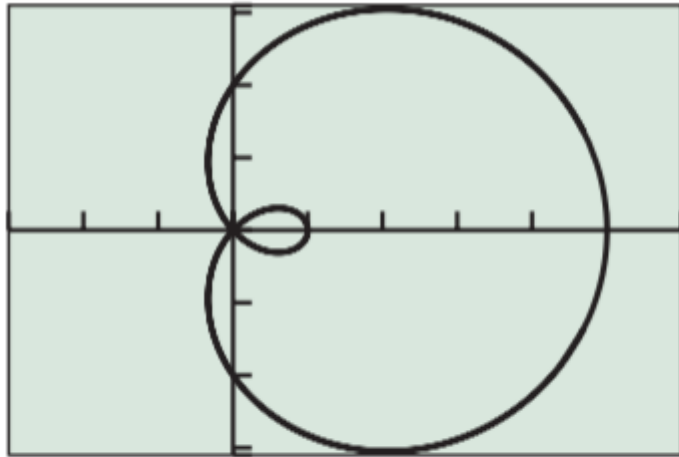
No asymptotes

Number of petals: n , if n is odd
 $2n$, if n is even

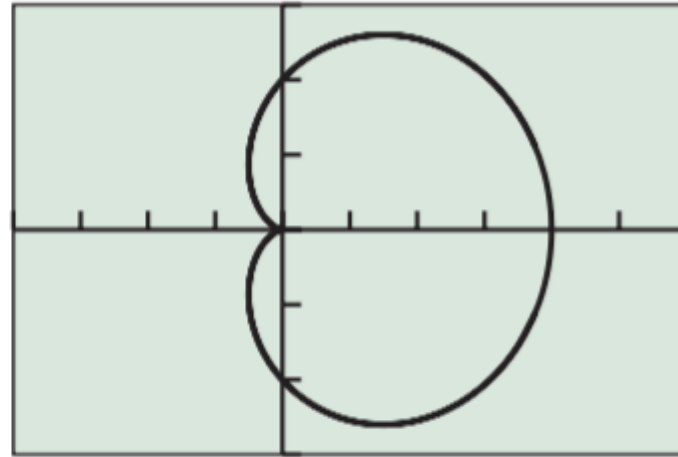
Limaçon Curves

The **limaçon curves** are graphs of polar equations of the form

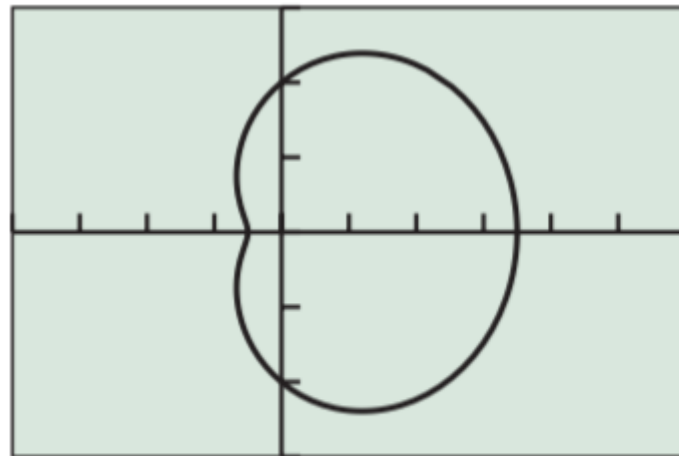
$$r = a \pm b \sin \theta \quad \text{and} \quad r = a \pm b \cos \theta$$



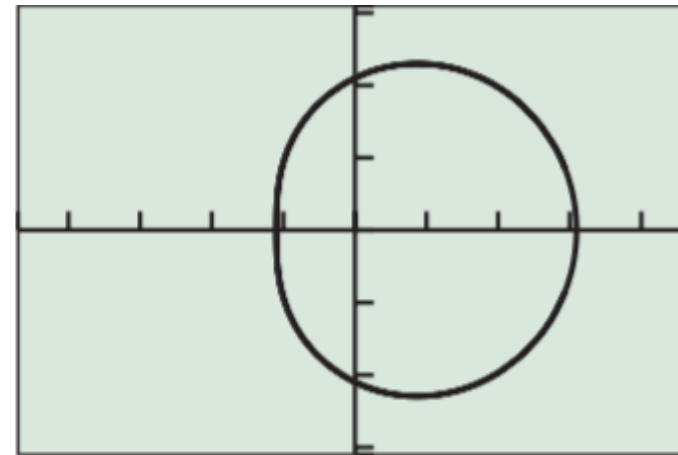
Limaçon with an inner loop: $\frac{a}{b} < 1$



Cardioid: $\frac{a}{b} = 1$



Dimpled limaçon: $1 < \frac{a}{b} < 2$



Convex limaçon: $\frac{a}{b} \geq 2$