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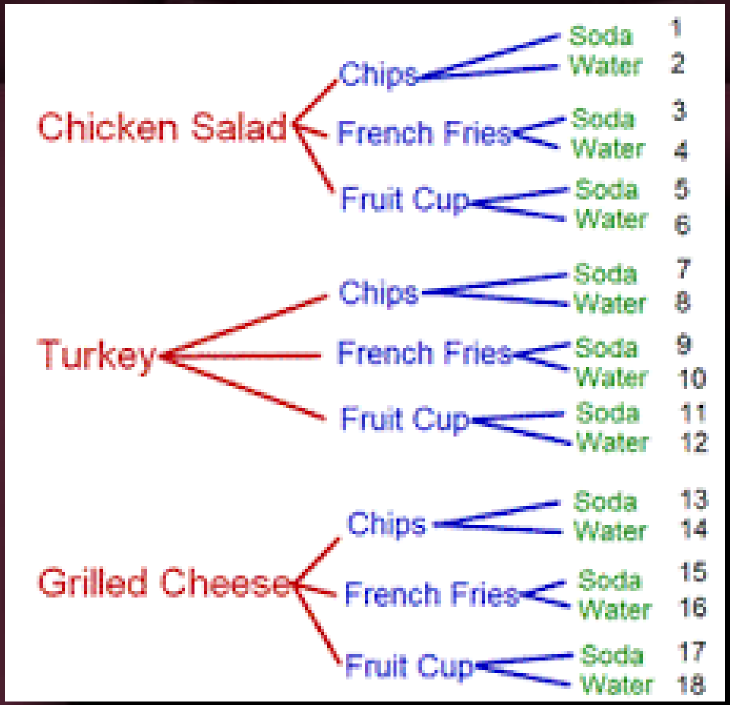

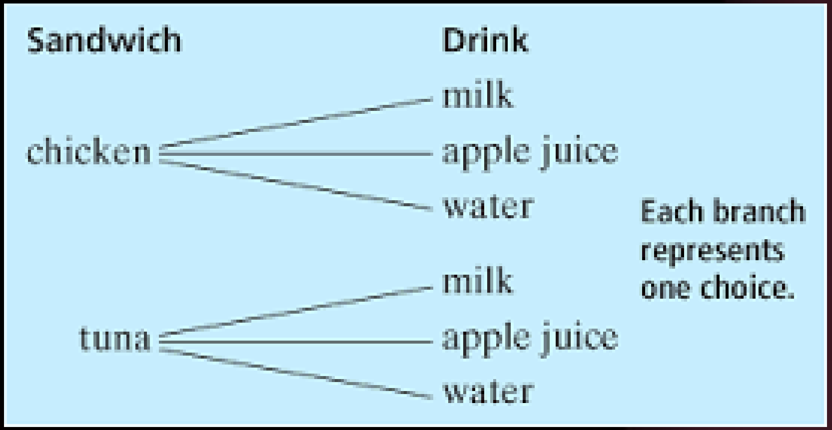

The Multiplication Principle of Counting

You would not want to draw the tree diagram for ordering five objects (*ABCDE*), but you should be able to see in your mind that it would have $5 \times 4 \times 3 \times 2 \times 1 = 120$ branches. A tree diagram is a geometric visualization of a fundamental counting principle known as the *Multiplication Principle*.

EXAMPLE 2 Using the Multiplication Principle

The Tennessee license plate shown here consists of three letters of the alphabet followed by three numerical digits (0 through 9). Find the number of different license plates that could be formed

(a) if there is no restriction on the letters or digits that can be used;

(b) if no letter or digit can be repeated.

Permutations

One important application of the Multiplication Principle of Counting is to count the number of ways that a set of n objects (called an ***n*-set**) can be arranged in order. Each such ordering is called a **permutation** of the set.

FACTORIALS

If n is a positive integer, the symbol $n!$ (read "n factorial") represents the product $n(n-1)(n-2)(n-3)\cdots 2 \bullet 1$. We also define $0! = 1$.

How many different outcomes are there for 9 people running the 100*m* race?

Permutations of an *n*-set

There are $n!$ permutations of an n-set.

Usually the elements of a set are distinguishable from one another, but we can adjust our counting when they are not, as we see in Example 3.

EXAMPLE 3 Distinguishable Permutations

Count the number of different 9-letter "words" (don't worry about whether they're in the dictionary) that can be formed using the letters in each word.

(a) DRAGONFLY **(b)** BUTTERFLY **(c)** BUMBLEBEE

Distinguishable Permutations

$$\frac{n!}{n_1!n_2!n_3! \cdots n_k!}.$$

permutations of n objects taken r at a time.

Permutation Counting Formula

The number of permutations of n objects taken r at a time is denoted ${}_nP_r$ and is given by

$$ {}_nP_r = \frac{n!}{(n-r)!} \quad \text{for} \quad 0 \le r \le n. $$

If $r > n$, then ${}_nP_r = 0$.

How many different outcomes for 1st, 2nd, and 3rd place are there for 9 people running the 100*m* race?

EXAMPLE 4 Counting Permutations

Evaluate each expression without a calculator.

(a) ${}_6P_4$ **(b)** ${}_{11}P_3$ **(c)** ${}_nP_3$

EXAMPLE 5 Applying Permutations

Sixteen actors answer a casting call to try out for roles as dwarfs in a production of *Snow White and the Seven Dwarfs*. In how many different ways can the director cast the seven roles?

Combinations

When we count permutations of n objects taken r at a time, we consider different orderings of the same r selected objects as being different permutations. In many applications we are only interested in the ways to *select* the r objects, regardless of the order in which we arrange them. These unordered selections are called **combinations of n objects taken r at a time**.

Combination Counting Formula

The number of combinations of n objects taken r at a time is denoted ${}_nC_r$ and is given by

$$ {}_nC_r = \frac{n!}{r!(n-r)!} \quad \text{for} \quad 0 \le r \le n. $$

If $r > n$, then ${}_nC_r = 0$.

A WORD ON NOTATION

Some textbooks use $P(n, r)$ instead of ${}_nP_r$ and $C(n, r)$ instead of ${}_nC_r$. Much more common is the notation $\binom{n}{r}$ for ${}_nC_r$. Both $\binom{n}{r}$ and ${}_nC_r$ are often read "n choose r."

EXAMPLE 6 Distinguishing Combinations from Permutations

In each of the following scenarios, tell whether permutations (ordered) or combinations (unordered) are being described.

(a) A president, vice-president, and secretary are chosen from a 25-member garden club.

(b) A cook chooses 5 potatoes from a bag of 12 potatoes to make a potato salad.

(c) A teacher makes a seating chart for 22 students in a classroom with 30 desks.

EXAMPLE 7 Counting Combinations

In the Miss America pageant, 51 contestants must be narrowed down to 10 finalists who will compete on national television. In how many possible ways can the ten finalists be selected?

EXAMPLE 8 Picking Lottery Numbers

The Georgia Lotto requires winners to pick 6 integers between 1 and 46. The order in which you select them does not matter; indeed, the lottery tickets are always printed with the numbers in ascending order. How many different lottery tickets are possible?

EXAMPLE 9 Selecting Pizza Toppings

Armando's Pizzeria offers patrons any combination of up to 10 different toppings: pepperoni, mushroom, sausage, onion, green pepper, bacon, prosciutto, black olive, green olive, and anchovies. How many different pizzas can be ordered

(a) if we can choose any three toppings?

(b) if we can choose any number of toppings (0 through 10)?

Formula for Counting Subsets of an *n*-Set

There are 2^n subsets of a set with n objects (including the empty set and the entire set).

EXAMPLE 10 Analyzing an Advertised Claim

A national hamburger chain used to advertise that it fixed its hamburgers "256 ways," since patrons could order whatever toppings they wanted. How many toppings must have been available?