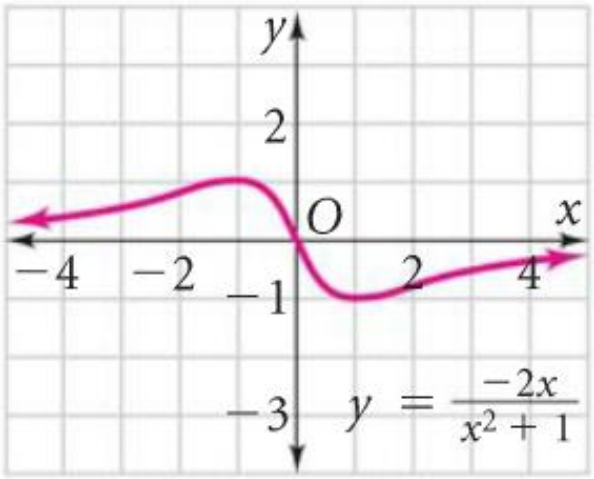
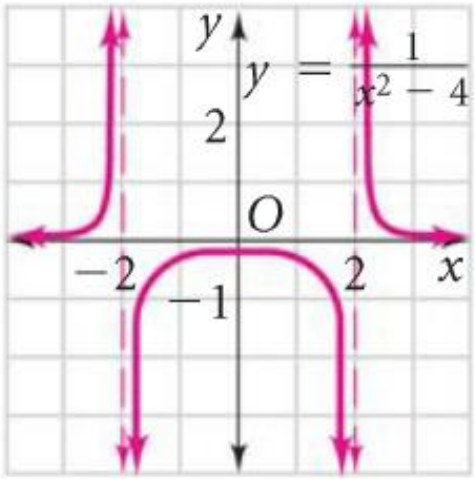
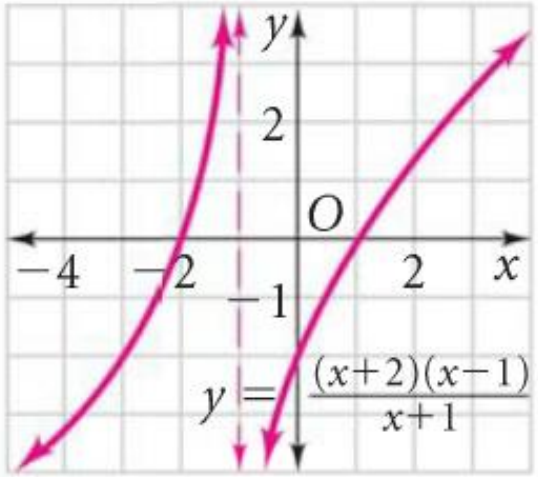
Compare the functions, denominators, asymptotes, and graphs below

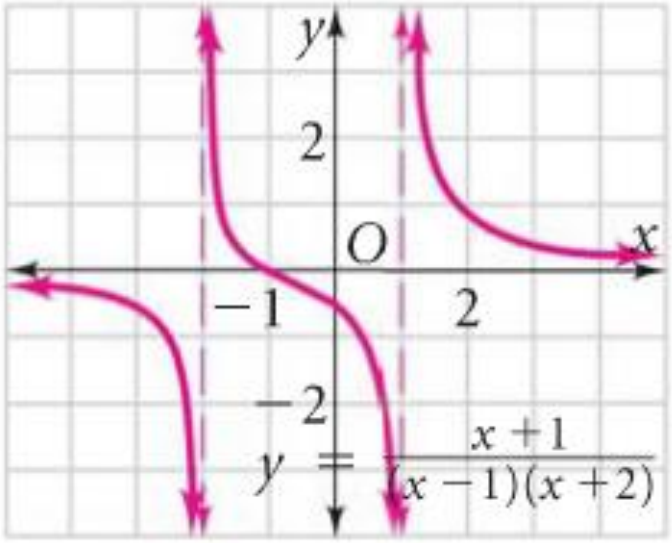
The graphs of the rational functions $y = \frac{-2x}{x^2 + 1}$, $y = \frac{1}{x^2 - 4}$, and $y = \frac{(x + 2)(x - 1)}{x + 1}$ are shown below.



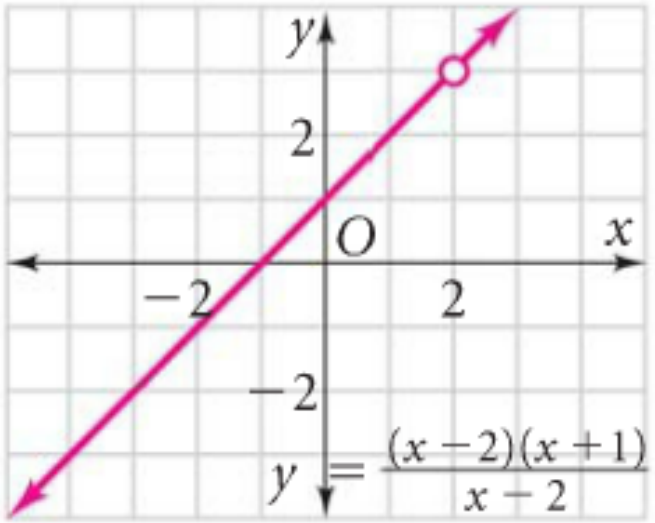




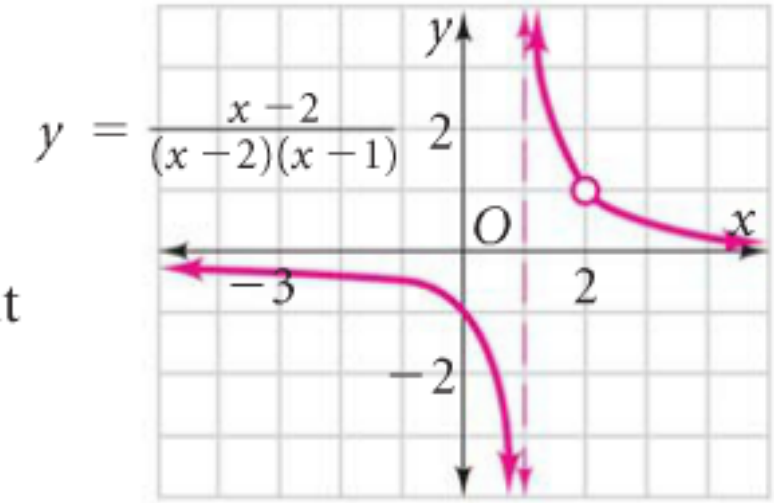
The graph of $y = \dfrac{x + 1}{(x - 1)(x + 2)}$ is shown at the right. The zeros of the denominator are 1 and -2. The graph has vertical asymptotes at those points.



2 is a zero of both the numerator and the denominator of the rational function $y = \frac{(x - 2)(x + 1)}{x - 2}$. The graph of this function is the same as the graph of $y = x + 1$, except it has a hole at $x = 2$.

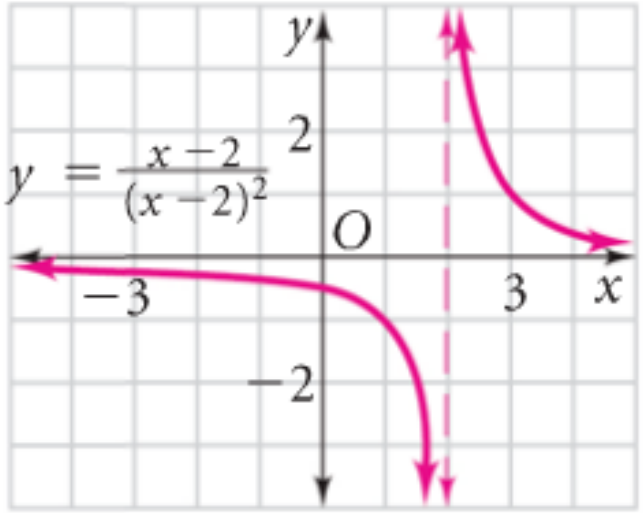


2 is a zero of both the numerator and the denominator of the rational function $y = \frac{x - 2}{(x - 2)(x - 1)}$. The graph of this function is the same as the graph of $y = \frac{1}{x - 1}$, except it has a hole at $x = 2$.



2 is a zero of both the numerator and the denominator of the rational function $y = \frac{x - 2}{(x - 2)^2}$.

The graph of this function is exactly the same as the graph of $y = \frac{1}{x - 2}$. The vertical asymptote is $x = 2$ and there is no hole.



Properties Vertical Asymptotes

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a vertical asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a, then there is a hole in the graph or a vertical asymptote at $x = a$.

EXAMPLE Finding Points of Discontinuity

For each rational function, find any points of discontinuity.

a. $y = \dfrac{1}{x^2 + 2x + 1}$

b. $y = \dfrac{-x + 1}{x^2 + 1}$

For each rational function, find any points of discontinuity.

a. $y = \dfrac{1}{x^2 - 16}$ **b.** $y = \dfrac{x^2 - 1}{x^2 + 3}$ **c.** $y = \dfrac{x + 1}{x^2 + 2x - 8}$

EXAMPLE Finding Vertical Asymptotes

Describe the vertical asymptotes and holes for the graph of each rational function.

a. $y = \dfrac{x + 1}{(x - 2)(x - 3)}$

b. $y = \dfrac{(x - 2)(x - 1)}{x - 2}$

c. $y = \dfrac{(x - 3)(x + 4)}{(x - 3)(x - 3)(x + 4)}$

Describe the vertical asymptotes and holes for the graph of each rational function.

a. $y = \frac{x - 2}{(x - 1)(x + 3)}$ **b.** $y = \frac{x - 2}{(x - 2)(x + 3)}$ **c.** $y = \frac{x^2 - 1}{x + 1}$

Assignment:

Find any points of discontinuity for each rational function.

1. $y = \dfrac{x + 3}{(x - 4)(x + 3)}$

2. $y = \dfrac{x - 2}{x^2 - 4}$

3. $y = \dfrac{(x - 3)(x + 1)}{(x - 2)}$

4. $y = \dfrac{3x(x + 2)}{x(x + 2)}$

5. $y = \dfrac{2}{(x + 1)}$

6. $y = \dfrac{4x}{x^3 - 9x}$

Describe the vertical asymptotes and holes for the graph of each rational function.

22. $y = \dfrac{x - 2}{(x + 2)(x - 2)}$

23. $y = -\dfrac{x}{x(x - 1)}$

24. $y = \dfrac{5 - x}{x^2 - 1}$

25. $y = \dfrac{x^2 - 2}{x + 2}$

26. $y = \dfrac{x^2 - 4}{x^2 + 4}$

27. $y = \dfrac{x + 3}{x^2 - 9}$