Properties

Horizontal Asymptotes

The graph of a rational function has at most one horizontal asymptote.

The graph of a rational function has a horizontal asymptote at $y = 0$ if the degree of the denominator is greater than the degree of the numerator.

$$f(x) = \frac{1}{x+2} \qquad f(x) = \frac{x+1}{x^2-1} \qquad f(x) = \frac{1}{(x-2)(x+3)}$$

If the degrees of the numerator and the denominator are equal, then the graph has a horizontal asymptote at $y = \frac{a}{b}$. a is the coefficient of the term of highest degree in the numerator and b is the coefficient of the term of highest degree in the denominator.

$$y = \frac{3x-5}{x} \qquad y = \frac{5x-1}{2x+3} \qquad y = \frac{-4x-3}{2x+1}$$

If the degree of the numerator is greater than the degree of the denominator, then the graph has no horizontal asymptote.

$$f(x) = \frac{x^2-1}{x+1} \qquad f(x) = \frac{x^3-27}{x-2}$$

Find the horizontal asymptote of $y = \frac{3x + 5}{x - 2}$.

Degree of numerator equals degree of denominator, therefore.....

Horizontal asymptote is $y = \frac{a}{b}$

$$y = \frac{3}{1}$$

$$y = 3$$

Find the horizontal asymptote of the graph of each rational function.

a. $y = \dfrac{-2x + 6}{x - 1}$

$y = -2$

b. $y = \dfrac{2x^2 + 5}{x^2 + 1}$

$y = 2$

c. $y = \dfrac{x + 2}{(x + 3)(x - 4)}$

$y = 0$

d. $y = \dfrac{x^2 + 6x - 27}{x - 2}$

There is NO Horizontal asymptote!