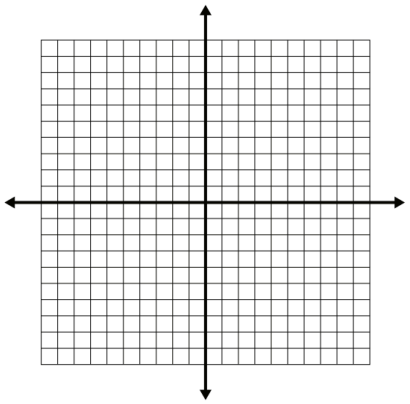
Name: _____ Date: _____ Period: _____

9.3 Rational Functions and Their Graphs

Part 1: Points of Discontinuity

1. Read the definition of a rational function. Focus on the last sentence, *"The domain of f(x) is all real numbers except those for which Q(x)=0."* Now read the definition of "point of discontinuity". Finding points of discontinuity focuses mostly on the denominator of the rational function.

2. Graph the following in desmos and add a table for points. $y = \frac{-2x}{x^2+1}$



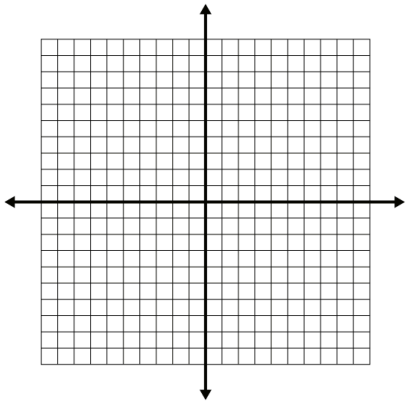
   a. Sketch it. Include any asymptotes and points.
   b. Are there any asymptotes? _____
   c. Are there any breaks in the graph? _____
   d. Is this graph continuous? _____
   e. Set the denominator equal to 0 and solve.

   f. What does your answer in part e mean after reading #1?

   _____

   _____

3. Graph the following in desmos and add a table for points. $y = \frac{1}{x^2-4}$



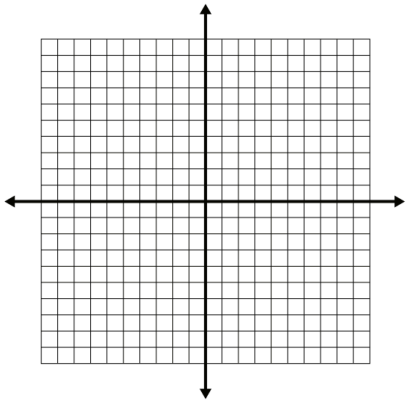
   a. Sketch it. Include any asymptotes and points.
   b. Are there any asymptotes? _____
   c. Are there any breaks in the graph? _____
   d. Is this graph continuous? _____
   e. Set the denominator equal to 0 and solve.

   f. What does your answer in part e mean after reading #1?

   _____

   _____

4. Graph the following in desmos and add a table for points. $y = \frac{(x+2)(x-1)}{x+1}$



   a. Sketch it. Include any asymptotes and points.
   b. Are there any asymptotes? _____
   c. Are there any breaks in the graph? _____
   d. Is this graph continuous? _____
   e. Set the denominator equal to 0 and solve.

   f. What does your answer in part e mean after reading #1?

   _____

   _____

5. How would you summarize the process of finding a point of discontinuity?

   _____

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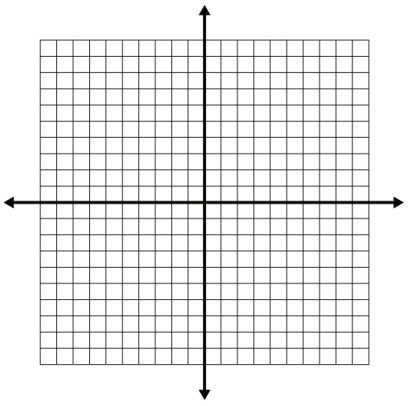
Name: _____ Date: _____ Period: _____

Part 2: Finding Vertical Asymptotes



1. Graph the following function in desmos. Sketch it. $y = \frac{x+1}{(x-1)(x+2)}$

   a. Set the denominator equal to zero and solve. _____
   b. Set the numerator equal to zero and solve. _____
   c. The solutions in **part a** represent the zeros of the denominator which are your vertical asymptotes. The answer in **part b** represents the x-intercept. Graph both parts.
   d. Foil the denominator. What do you notice about the degrees of the numerator and denominator? Did any binomial(s) cancel out?

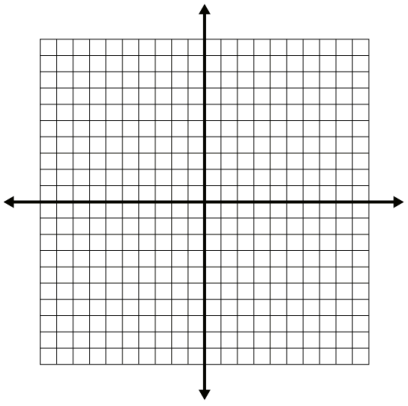
   _____

   _____

   e. Read the properties of vertical asymptotes. What properties are illustrated here?

   _____

   _____



2. Graph the following function in desmos. Sketch it. $y = \frac{(x-2)(x+1)}{(x-2)}$

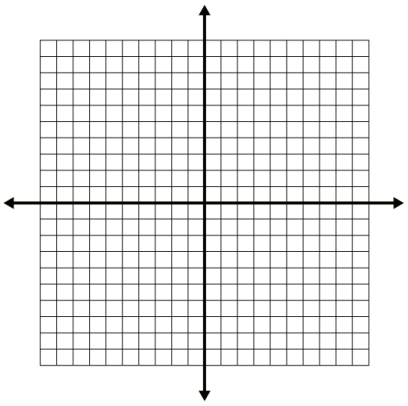
   a. Set the denominator equal to zero and solve. _____
   b. Set the numerator equal to zero and solve. _____
   c. Simplify the function. What do you get?_____
   d. Since the graph simplifies, the solutions in **part a** represent the zeros of the denominator which are your holes. The answer in **part b** represents the x-intercept. Graph both parts.
   e. Foil the numerator. What do you notice about the degrees of the numerator and denominator? _____

      _____

   f. Read the properties of vertical asymptotes. What properties are illustrated here?

   _____

   _____



3. Graph the following function in desmos. Sketch it. $y = \frac{(x-2)}{(x-2)(x-1)}$

   a. Set the denominator equal to zero and solve. _____
   b. Set the numerator equal to zero and solve. _____
   c. Simplify the function. What do you get?_____
   d. Since the graph simplifies, the solutions in **part a** represent the a hole and a vertical asymptote. The answer in **part b** represents the hole since the graph simplifies. *A hole is a value that is a "zero" for both the numerator and denominator.* Graph both parts and convert it to a table. To graph a hole put an empty point at that value. What do you notice about x=2 in the table?

      _____

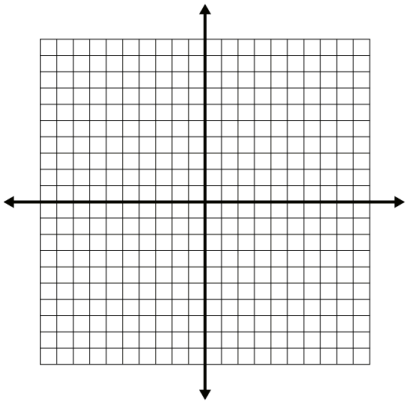
   e. Read the properties of vertical asymptotes. What properties are illustrated here?

   _____

   _____

Name: _____ Date: _____ Period: _____

4. Graph the following function in desmos. Convert it to a table. Sketch it. $y = \frac{x-2}{(x-2)^2}$



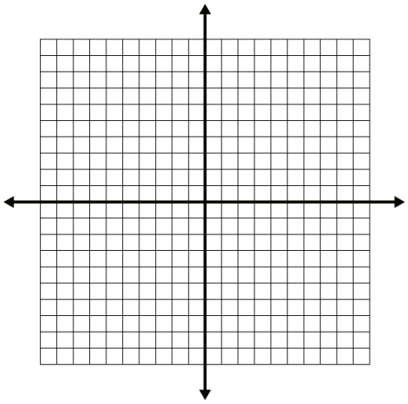
   a. Set the denominator equal to zero, solve and sketch it. _____
   b. Set the numerator equal to zero, solve, and plot it. _____
   c. Simplify the function. What cancels out? What do you get?_____
   d. Since the graph simplifies, the common factor is a hole. *A hole is a value that is a "zero" for both the numerator and denominator.* To graph a hole, set the factor equal to zero, solve, substitute it into the simplified function to generate the hole's y-coordinate and plot an empty point at that value. Check using the table.
   e. What properties of vertical asymptotes are illustrated here?

_____

_____

Part 3: Finding Horizontal Asymptotes

5. Graph the following function in desmos. Convert it to a table. $y = \frac{3x+5}{x-2}$



   a. Set the denominator equal to zero, solve and sketch it. _____
   b. Set the numerator equal to zero, solve and plot it. _____
   c. Simplify the function by using long division. What do you get? _____
   d. What is the horizontal asymptote of **part c**? Sketch it. _____
   e. Compare the degree of the numerator to the degree of the denominator. What do you notice? _____
   f. Based on the properties of horizontal asymptotes and your answer in **part e,** what conclusion(s) can you make?

_____

_____

   g. Using the table, pick enough points to the left and right of the vertical asymptote(s) to generate a trend for each branch. Plot and sketch your graph.

6. Consider the problems from Part 2.
   a. Part 2 #1:
      i. Foil the denominator. _____
      ii. Compare the degrees of the numerator and denominator. What do you notice? _____
      iii. Based on your answer in 6aii and the properties of horizontal asymptotes, what conclusion can you make? _____
   b. Part 2 #2:
      i. Foil the numerator. _____
      ii. Compare the degrees of the numerator and denominator. What do you notice? _____
      iii. Based on your answer in 6bii and the properties of horizontal asymptotes, what conclusion can you make? _____
   c. Part 2 #3:
      i. Foil the denominator. _____
      ii. Compare the degrees of the numerator and denominator. What do you notice? _____
      iii. Based on your answer in 6cii and the properties of horizontal asymptotes, what conclusion can you make? _____
   d. Part 2 #4:
      i. Foil the numerator. _____
      ii. Compare the degrees of the numerator and denominator. What do you notice? _____
      iii. Based on your answer in 6dii and the properties of horizontal asymptotes, what conclusion can you make? _____

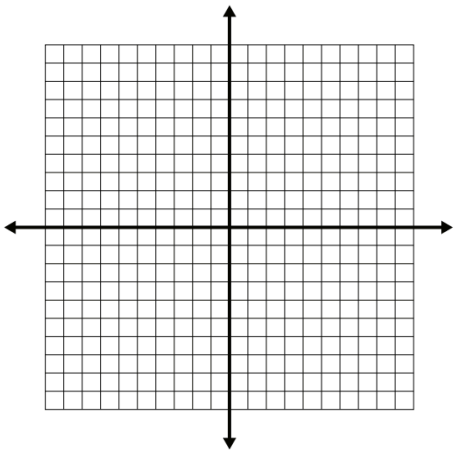
Name: _____ Date: _____ Period: _____

Part 4: Curve Sketching

You will use all the skills and strategies presented in the previous parts to sketch the graph of a rational function.

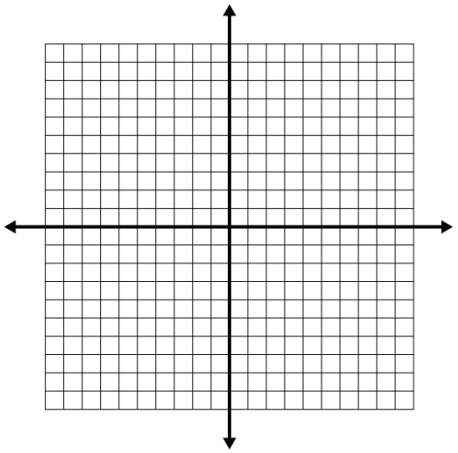
1. Consider $y = \dfrac{(x+6)}{(x-2)(x+3)}$

   a. Find any vertical asymptote(s). Sketch them. _____
   b. Find any horizontal asymptotes(s) Sketch them. _____
   c. Find any holes. Sketch them. _____
   d. Find any x-intercepts. Sketch them. _____
   e. Find any y-intercepts. Sketch them. _____
   f. Pick enough points to the left and right of any vertical asymptotes or holes to generate points to plot. Use your table to help you. Connect them.
   g. Graph the function in desmos and compare.



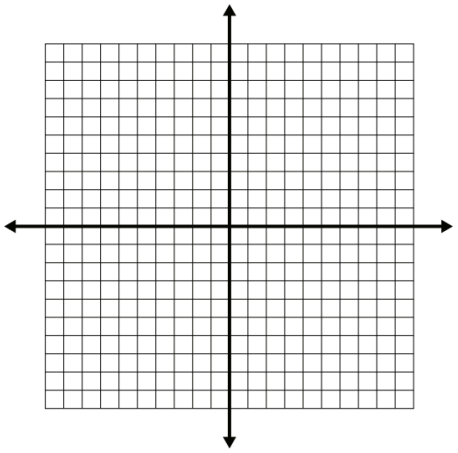
2. Consider $y = \dfrac{-4x}{x^3 - 4x}$

   a. Find any vertical asymptote(s). Sketch them. _____
   b. Find any horizontal asymptotes(s) Sketch them. _____
   c. Find any holes. Sketch them. _____
   d. Find any x-intercepts. Sketch them. _____
   e. Find any y-intercepts. Sketch them. _____
   f. Pick enough points to the left and right of any vertical asymptotes or holes to generate points to plot. Use your table to help you. Connect them.
   g. Graph the function in desmos and compare.



3. Consider $y = \dfrac{x(x+1)}{x+1}$

   a. Find any vertical asymptote(s). Sketch them. _____
   b. Find any horizontal asymptotes(s) Sketch them. _____
   c. Find any holes. Sketch them. _____
   d. Find any x-intercepts. Sketch them. _____
   e. Find any y-intercepts. Sketch them. _____
   f. Pick enough points to the left and right of any vertical asymptotes or holes to generate points to plot. Use your table to help you. Connect them.
   g. Graph the function in desmos and compare.



4. Consider $y = \dfrac{x+4}{x-4}$

   a. Find any vertical asymptote(s). Sketch them. _____
   b. Find any horizontal asymptotes(s) Sketch them. _____
   c. Find any holes. Sketch them. _____
   d. Find any x-intercepts. Sketch them. _____
   e. Find any y-intercepts. Sketch them. _____
   f. Pick enough points to the left and right of any vertical asymptotes or holes to generate points to plot. Use your table to help you. Connect them.
   g. Graph the function in desmos and compare.