Summation Notation

We want to look at the formulas for summing the terms of arithmetic and geometric sequences, but first we need a notation for writing the sum of an indefinite number of terms. The capital Greek letter sigma (Σ) provides our shorthand notation for a "summation."

DEFINITION Summation Notation

In **summation notation**, the sum of the terms of the sequence $\{a_1, a_2, \ldots, a_n\}$ is denoted

last term

explicit formula

$$\sum_{k=1}^{n} a_k$$

1st term

which is read "the sum of a_k from $k = 1$ to n."

The variable k is called the **index of summation**.

$$2+4+6+8+10 = \sum_{k=1}^{5} 2k = 30$$

EXPLORATION 1 Summing with Sigma

Sigma notation is actually even more versatile than the definition above suggests. See if you can determine the number represented by each of the following expressions.

1. $\sum_{k=1}^{5} 3k$ **2.** $\sum_{k=5}^{8} k^2$ **3.** $\sum_{n=0}^{12} \cos(n\pi)$

1) $3+6+9+12+15=45$

2) $25+36+49+64=174$

3.) $1+(-1)+1+(-1)+1+(-1)+1+(-1)+1+(-1)+1+(-1)+1$

THEOREM Sum of a Finite Arithmetic Sequence

Let $\{a_1, a_2, a_3, \ldots, a_n\}$ be a finite arithmetic sequence with common difference d. Then the sum of the terms of the sequence is

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$

$$= n\left(\frac{a_1 + a_n}{2}\right) = 5\left(\frac{2+10}{2}\right) = 5 \cdot 6 = 30$$

$$= \frac{n}{2}(2a_1 + (n-1)d)$$

EXAMPLE 1 Summing the Terms of an Arithmetic Sequence

A corner section of a stadium has 8 seats along the front row. Each successive row has two more seats than the row preceding it. If the top row has 24 seats, how many seats are in the entire section?

$a_1 = 8$

$a_n = 24$

$a_n = a_1 + (n-1)d$

$24 = 8 + (n-1)2$

$\frac{16}{2} = \frac{\cancel{2}(n-1)}{\cancel{2}}$

$8 = n-1$

$n = 9$ rows

$\frac{n}{2}(a_1 + a_n)$

$\frac{9}{2}(8+24)$

$\frac{9}{\cancel{2}}(\cancel{32}^{16}) = 9 \cdot 16 = 144$ seats

THEOREM **Sum of a Finite Geometric Sequence**

Let $\{a_1, a_2, a_3, \ldots, a_n\}$ be a finite geometric sequence with common ratio $r \neq 1$.

Then the sum of the terms of the sequence is

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$$
$$= \frac{a_1(1 - r^n)}{1 - r}$$

$3 + 6 + 12 + 24 + 48 + 96$

$$= \frac{3(1-(2)^6)}{1-2} = \frac{3(1-2^6)}{-1} = \boxed{189}$$

EXAMPLE 2 Summing the Terms of a Geometric Sequence

Find the sum of the geometric sequence 4, −4/3, 4/9, −4/27, . . . , $4(-1/3)^{10}$.

$r = -\frac{1}{3}$

$a_n = a_1 r^{(n-1)}$

$10 = n - 1$

$n = 11$

$$\frac{a_1(1 - r^n)}{1 - r} = \frac{4\left(1 - \left(-\frac{1}{3}\right)^{11}\right)}{1 - \left(-\frac{1}{3}\right)}$$

$$= \frac{4\left(1 - \left(-\frac{1}{3}\right)^{11}\right)}{\frac{4}{3}}$$