EXAMPLE 4 Finding a Derivative at a Point

Find $f'(4)$ if $f(x) = 2x^2 - 3$.

$f(4) = 2(4)^2 - 3$
$= 32 - 3 = 29$

SOLUTION

$a = 4$

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

$$f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(4) = \lim_{x \to 4} \frac{2x^2 - 3 - 29}{x - 4} = \lim_{x \to 4} \frac{2x^2 - 32}{x - 4}$$

$$= \lim_{x \to 4} \frac{2(x^2 - 16)}{x - 4}$$

$$= \lim_{x \to 4} \frac{2(x+4)\cancel{(x-4)}}{\cancel{x-4}}$$

$$= \lim_{x \to 4} 2(x+4) = 2(4+4)$$
$$= 16$$

$$\boxed{f'(4) = 16}$$

EXAMPLE 5 Finding the Derivative of a Function

(a) Find $f'(x)$ if $f(x) = x^2$.

$f(4)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + \cancel{h}^{0})}{\cancel{h}} = 2x$$

$$\boxed{f'(x) = 2x}$$

To emphasize the connection with slope $\Delta y/\Delta x$, Leibniz used the notation dy/dx for the derivative. (The dy and dx were his "ghosts of departed quantities.") This **Leibniz notation** has several advantages over the "prime" notation, as you will learn when you study calculus. We will use both notations in our examples and exercises.

(b) Find $\dfrac{dy}{dx}$ if $y = \dfrac{1}{x}$.

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x}{x}\cdot\frac{1}{(x+h)} - \frac{1}{x}\cdot\frac{(x+h)}{(x+h)}}{h}$$

$$\lim_{h\to 0} \frac{x-(x+h)}{x(x+h)}\cdot\frac{1}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x}-\cancel{x}-\cancel{h}}{x\cancel{h}(x+h)} = \lim_{h\to 0}\frac{-1}{x(x+\cancel{h})}$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{x^2}}$$