The Law of Sines

Law of Sines

In any $\triangle ABC$ with angles A, B, and C opposite sides a, b, and c, respectively, the following equation is true:

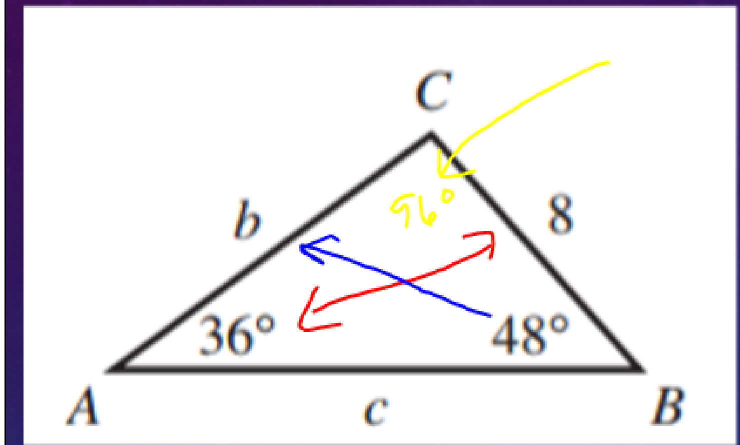
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

ratios

Solving Triangles (AAS, ASA)

EXAMPLE 1 **Solving a Triangle Given Two Angles and a Side**

Solve $\triangle ABC$ given that $\angle A = 36°$, $\angle B = 48°$, and $a = 8$.



$$\angle C = 180° - 36° - 48° = 96°$$

$$c = 13.54$$

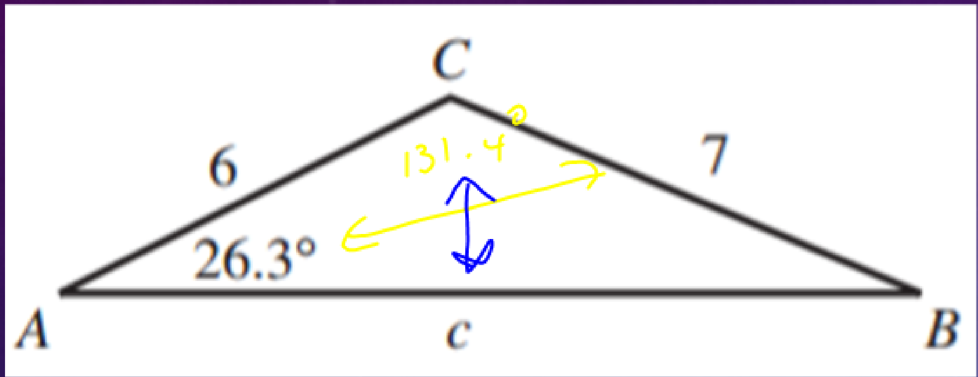
$$\boxed{\frac{\sin 36°}{8}} = \frac{\sin 48°}{b}$$

$$\frac{\sin 96°}{c} = \frac{\sin 36°}{8}$$

$$\frac{b\,\cancel{\sin 36°}}{\cancel{\sin 36}} = \frac{8 \sin 48°}{\sin 36°} = 10.11$$

EXAMPLE 2 Solving a Triangle Given Two Sides and an Angle

Solve $\triangle ABC$ given that $a = 7$, $b = 6$, and $\angle A = 26.3°$.



$$\frac{\sin 26.3°}{7} = \frac{\sin B}{6}$$

$$\frac{\cancel{7} \sin B}{\cancel{7}} = \frac{6 \sin 26.3°}{7}$$

$$\angle B = \sin^{-1}\left[\frac{6 \sin 26.3°}{7}\right]$$

$$\angle B = 22.3°$$

$$\angle C = 180° - 26.3° - 22.3°$$

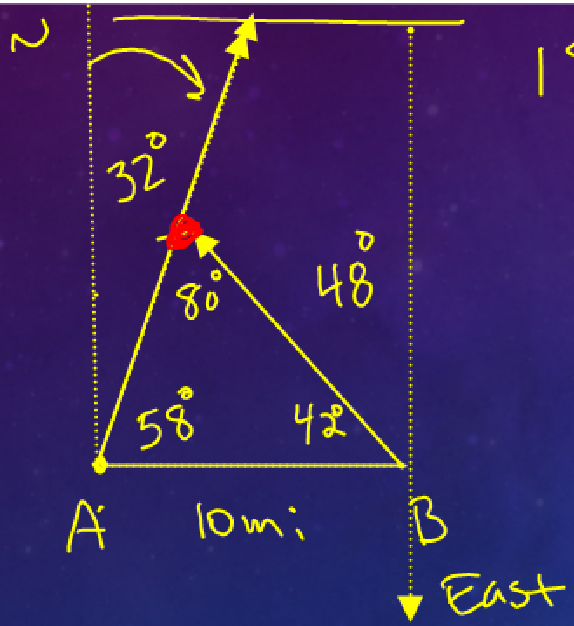
$$\angle C = 131.4°$$

$$\frac{\sin 131.4°}{c} = \frac{\sin 26.3°}{7}$$

$$c = 11.45$$

EXAMPLE 4 Locating a Fire

Forest Ranger Chris Johnson at ranger station *A* sights a fire in the direction 32° east of north. Ranger Rick Thorpe at ranger station *B*, 10 miles due east of *A*, sights the same fire on a line 48° west of north. Find the distance from each ranger station to the fire.



$180-58-42=80$

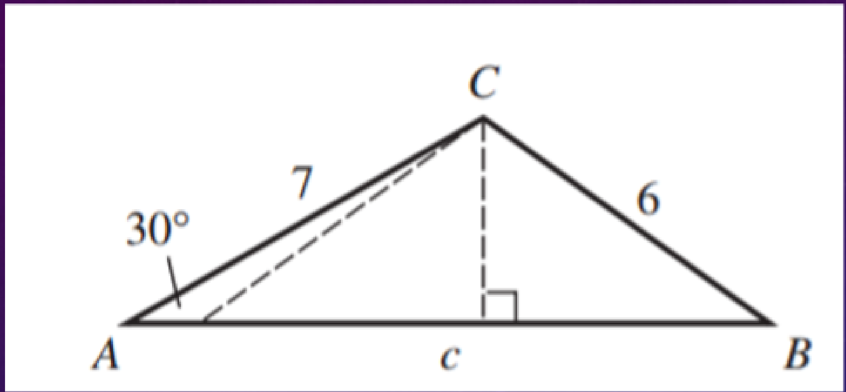
$\frac{\sin 80°}{10}=\frac{\sin 58°}{a}$ $\quad a=8.6$

$\frac{\sin 80°}{10}=\frac{\sin 42°}{b}$ $\quad b=6.8$

EXAMPLE 3 Handling the Ambiguous Case

Solve $\triangle ABC$ given that $a = 6$, $b = 7$, and $\angle A = 30°$.

SSA



$\dfrac{\sin 30°}{1} = \dfrac{h}{7}$

$h = 7 \sin 30° = 3.5$

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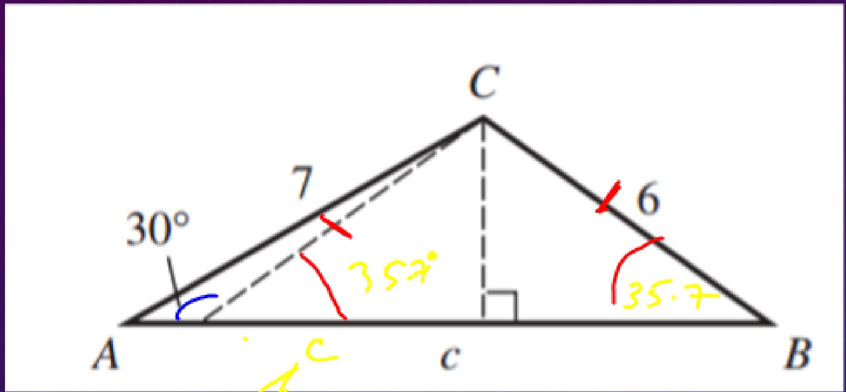
Solve $\triangle ABC$ given that $a = 6$, $b = 7$, and $\angle A = 30°$.



$$\frac{\sin 30°}{6} = \frac{\sin B}{7}$$

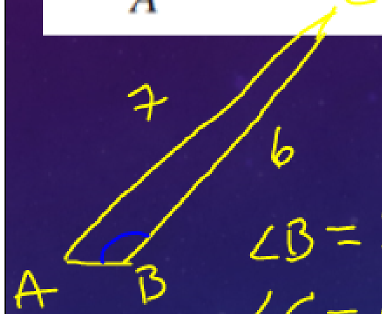
$$\sin B = \frac{7 \sin 30°}{6}$$

$$\angle B = \sin^{-1}\left(\frac{7 \sin 30°}{6}\right) = 35.7°$$

$$\angle C = 180 - 30 - 35.7° = 114.3°$$

$$\frac{\sin 114.3°}{c} = \frac{\sin 30°}{6}$$

$$c = 10.9$$



$$\frac{\sin 5.7°}{c} = \frac{\sin 30°}{6}$$

$$c = 1.2$$

$$\angle B = 180 - 35.7° = 144.3°$$

$$\angle C = 180° - 144.3° - 30° = 5.7°$$