

G of X

Recognizing Functions and Function Families

Warm Up

Identify the domain and range of the relation described by each set of ordered pairs. Write an equation using the variables x and y that could map the domain to the range.

1. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2)\}$
2. $\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$
3. $\{(-3, 3), (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

Learning Goals

- Define a function as a relation that assigns each element of the domain to exactly one element of the range.
- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

Key Terms

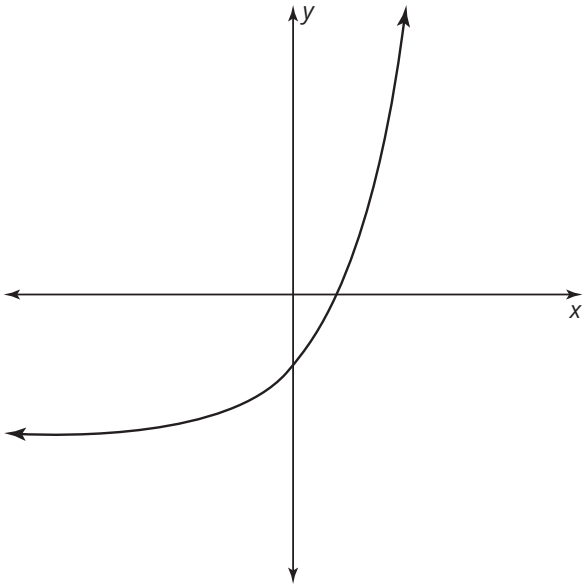
- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- linear absolute value functions
- linear piecewise functions
- x -intercept
- y -intercept

You have sorted graphs by their graphical behaviors. How can you describe the common characteristics of the graphs of the functions?

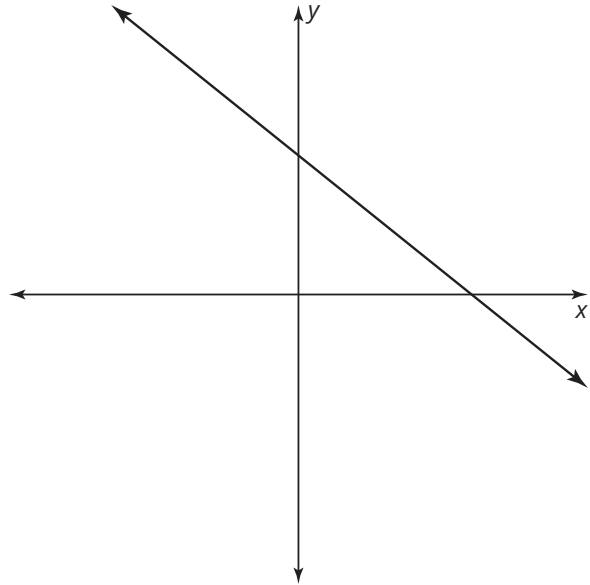
Odd One Out

- Which of the graphs shown does not belong with the others?
Explain your reasoning.

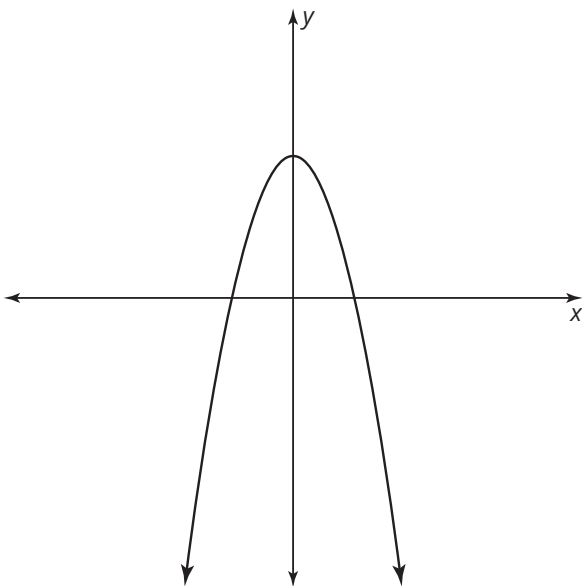
Graph A



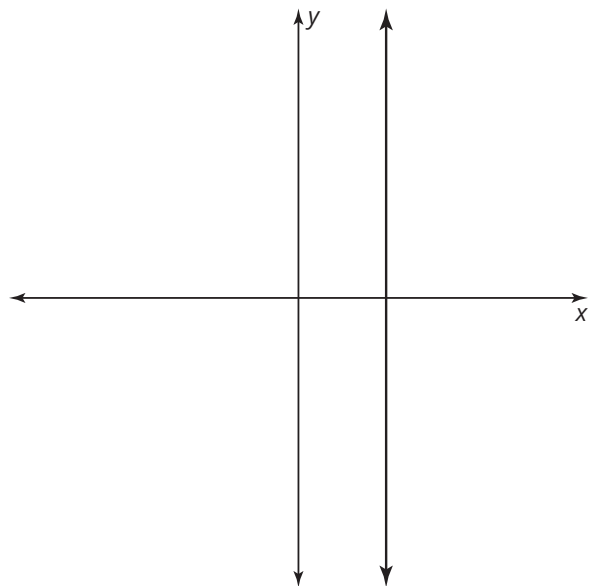
Graph B



Graph C



Graph D





A relation can be represented in the following ways.

Ordered Pairs

$\{(-2, 2), (0, 2), (3, -4), (3, 5)\}$

Equation

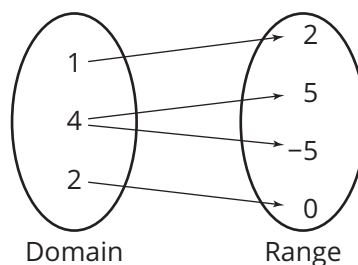
$$y = \frac{2}{3}x - 1$$

A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

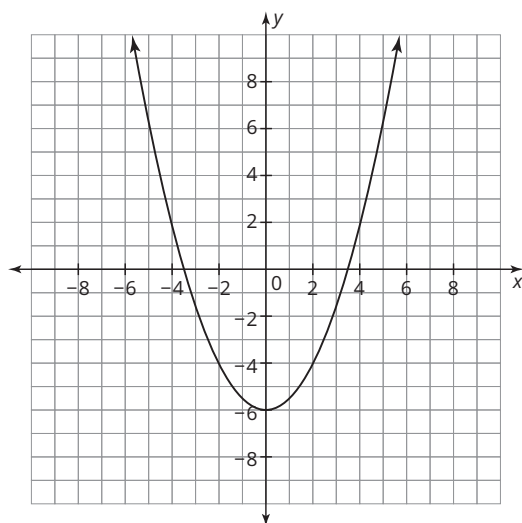
Verbal

The relation between students in your school and each student's birthday.

Mapping



Graph



Table

Domain	Range
-1	1
2	0
5	-5
6	-5
7	-8



So all functions are relations, but only some relations are functions.

A **function** is a relation that assigns to each element of the domain exactly one element of the range. Functions can be represented in a number of ways.

1. **Analyze the relation represented as a table. Is the relation a function? Explain your reasoning.**
2. **Analyze the relation represented as a mapping. Is the relation a function? Explain your reasoning.**
3. **Analyze the relation represented verbally. Is the relation a function? Explain your reasoning.**

You can write an equation representing a function using *function notation*. Let's look at the relationship between an equation and function notation.

Consider this scenario. U.S. Shirts charges \$8 per shirt plus a one-time charge of \$15 to set up a T-shirt design.

The equation $y = 8x + 15$ can be written to model this situation. The independent variable x represents the number of shirts ordered, and the dependent variable y represents the total cost of the order, in dollars.

This is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it. Because this relationship is a function, you can write $y = 8x + 15$ in function notation.

$f(x) = 8x + 15$

name of function

independent variable

The cost, defined by f , is a function of x , defined as the number of shirts ordered.

You can write a function in a number of different ways. You could write the T-shirt cost function as $C(s) = 8s + 15$, where the cost, defined as C , is a function of s , the number of shirts ordered.

4. Consider the U.S. Shirts function, $C(s) = 8s + 15$. What expression in the function equation represents:

a. the domain of the function?

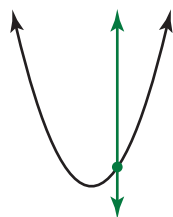
b. the range of the function?

5. Describe the possible domain and range for this situation.

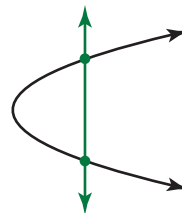
Function notation is a way of representing functions algebraically. The function notation $f(x)$ is read as “ f of x ” and indicates that x is the independent variable.

If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x .

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.



function



not a function

A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

The Vertical Line Test applies for both *discrete* and *continuous graphs*.

6. How can you determine if a relation represented as ordered pairs is a function? Explain your reasoning.

7. How can you determine if a relation represented as an equation is a function? Explain your reasoning.

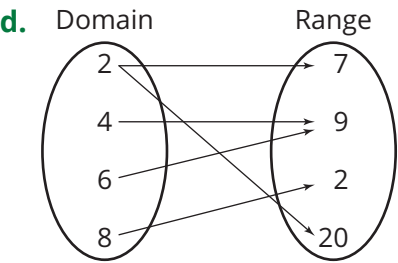
8. Determine which relations represent functions. If the relation is not a function, state why not.

a. $y = 3^x$

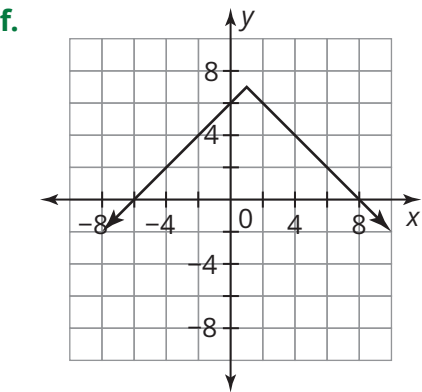
b. For every house, there is one and only one street address.

c.

Domain	Range
-1	4
0	0
3	-2
0	4



e. $\{(-7, 5), (-5, 5), (2, -2), (3, 5)\}$



9. Analyze the three graphs Judy grouped together in the previous lesson, graphs D, K, and O. Are the graphs she grouped functions? Explain your conclusion.

10. Use the Vertical Line Test to sort the graphs in the previous lesson into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.

Functions	Non-Functions

ACTIVITY
3.2

Domain and Range of a Function



You have identified the domain and range of a function given its equation.

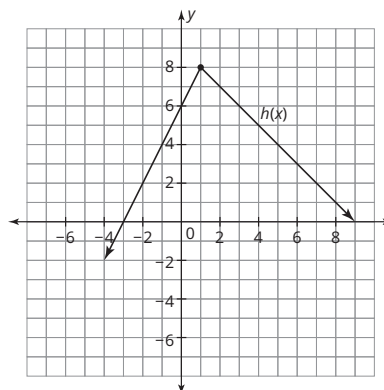
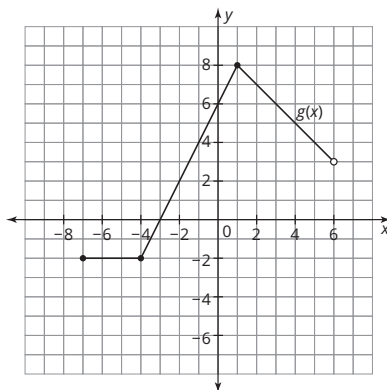
1. Explain how you can identify the domain and range of a function given:

a. a verbal statement.

b. a graph.

Worked Example

There are different ways to write the domain and range of a function given its graph.



	Domain		Range	
	$g(x)$	$h(x)$	$g(x)$	$h(x)$
In Words	The domain is all real numbers greater than or equal to -7 and less than 6 .	The domain is the set of all real numbers.	The range is all real numbers greater than or equal to -2 and less than or equal to 8 .	The range is all real numbers less than or equal to 8 .
Using Notation	$-7 \leq x < 6$	$-\infty < x < \infty$	$-2 \leq y \leq 8$	$y \leq 8$

2. Consider the Graph Cards from the previous activity that include continuous functions. Label each of these cards with the appropriate domain and range.

ACTIVITY
3.3

Linear, Constant, and Exponential Functions



When determining whether a graph is increasing or decreasing, read the graph from left to right.

Gather all of the graphs that you identified as functions.

A function is described as increasing when the value of the dependent variable increases as the value of the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

A function is described as decreasing when the value of the dependent variable decreases as the value of the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the value of the dependent variable of a function remains constant over the entire domain, then the function is called a **constant function**.

1. Analyze each graph from left to right. Sort all the graphs into one of the four groups listed.

- **increasing function**
- **decreasing function**
- **constant function**
- **a combination of increasing, decreasing, and constant**

Record the function letter in the appropriate column of the table shown.

Increasing Function	Decreasing Function	Constant Function	Combination of Increasing, Decreasing, and Constant Functions

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Use graphing technology to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x$
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = \left(\frac{1}{2}\right)^x - 5$
- $f(x) = 2$, where x is an integer
- $f(x) = 2^x$, where x is an integer
- $f(x) = -x + 3$, where x is an integer

3. Consider the six graphs and functions that are increasing functions, decreasing functions, or constant functions.
- a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2

- b. What is the same about all the functions in each group?

Think

about:

Be sure to correctly interpret the domain of each function. Also, remember to use parentheses when entering fractions into your calculator.

Ask

yourself:

What other variables have you used to represent a linear function?

You have just sorted the graphs into their own *function families*. A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a , b , and c are real numbers, and b is greater than 0 but not equal to 1.

4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. If $f(x) = ax + b$ represents a linear function, describe the a and b values that produce a constant function.

Place these two groups of graphs off to the side. You will need them again.



A function has an **absolute minimum** if there is a point on the graph of the function that has a y -coordinate that is less than the y -coordinate of every other point on the graph. A function has an **absolute maximum** if there is a point on the graph of the function that has a y -coordinate that is greater than the y -coordinate of every other point on the graph.

1. Sort the graphs from the combination of increasing, decreasing, and constant category in the previous activity into one of the three groups listed.

- those that have an absolute minimum value
- those that have an absolute maximum value
- those that have no absolute minimum or maximum value

Then record the function letter in the appropriate column of the table shown.

Absolute Minimum	Absolute Maximum	No Absolute Minimum or Absolute Maximum

2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Use graphing technology to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x^2 + 8x + 12$ • $f(x) = -|x|$
- $f(x) = |x - 3| - 2$ • $f(x) = -3x^2 + 4$, where x is integer
- $f(x) = x^2$ • $f(x) = -\frac{1}{2}x^2 + 2x$
- $f(x) = |x|$ • $f(x) = -2|x + 2| + 4$

3. Consider the graphs of functions that have an absolute minimum or an absolute maximum.

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2

b. What is the same about all the functions in each group?

You have just sorted functions into two more function families.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a , b , and c are real numbers, and a is not equal to 0.

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a , b , and c are real numbers, and a is not equal to 0.

4. Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions.



ACTIVITY

3.5

Linear Piecewise Functions



Analyze each of the functions shown. These functions represent the last two graphs of functions from the no absolute minimum and no absolute maximum category.

$$f(x) = \begin{cases} -2, & x < 0 \\ \frac{1}{2}x - 2, & x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} -2x + 10, & x < 3 \\ 4, & 3 \leq x < 7 \\ -2x + 18, & x \geq 7 \end{cases}$$

1. Use graphing technology to determine the shapes of their graphs. Then match each function to its corresponding graph by writing the function directly on the graph that it represents.

You have just sorted the remaining functions into one more function family. The family of **linear piecewise functions** includes functions that have equation changes for different parts, or pieces, of the domain.





You have now sorted each of the graphs and equations representing functions into one of five function families: linear, exponential, quadratic, linear absolute value, and linear piecewise. Let's now focus on linear and exponential functions, which are the functions that you will explore in-depth in this course.

- 1. Glue your sorted graphs and functions to the appropriate function family graphic organizer located at the end of the lesson. Write a description of the graphical behavior for each function family.**

Hang on to your graphic organizers. They will be a great resource moving forward!

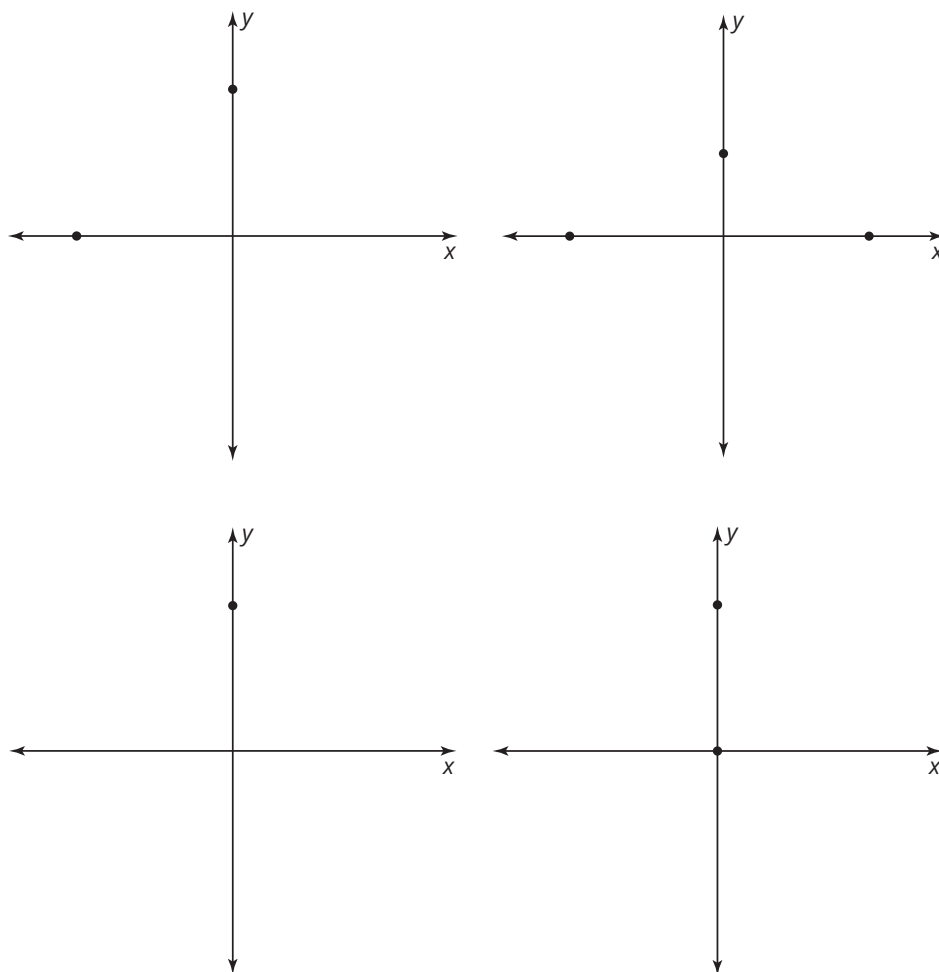
Don't worry—you don't need to know everything there is to know about these function families right now. As you progress through this course, you will learn more about linear and exponential function families.

TALK the TALK

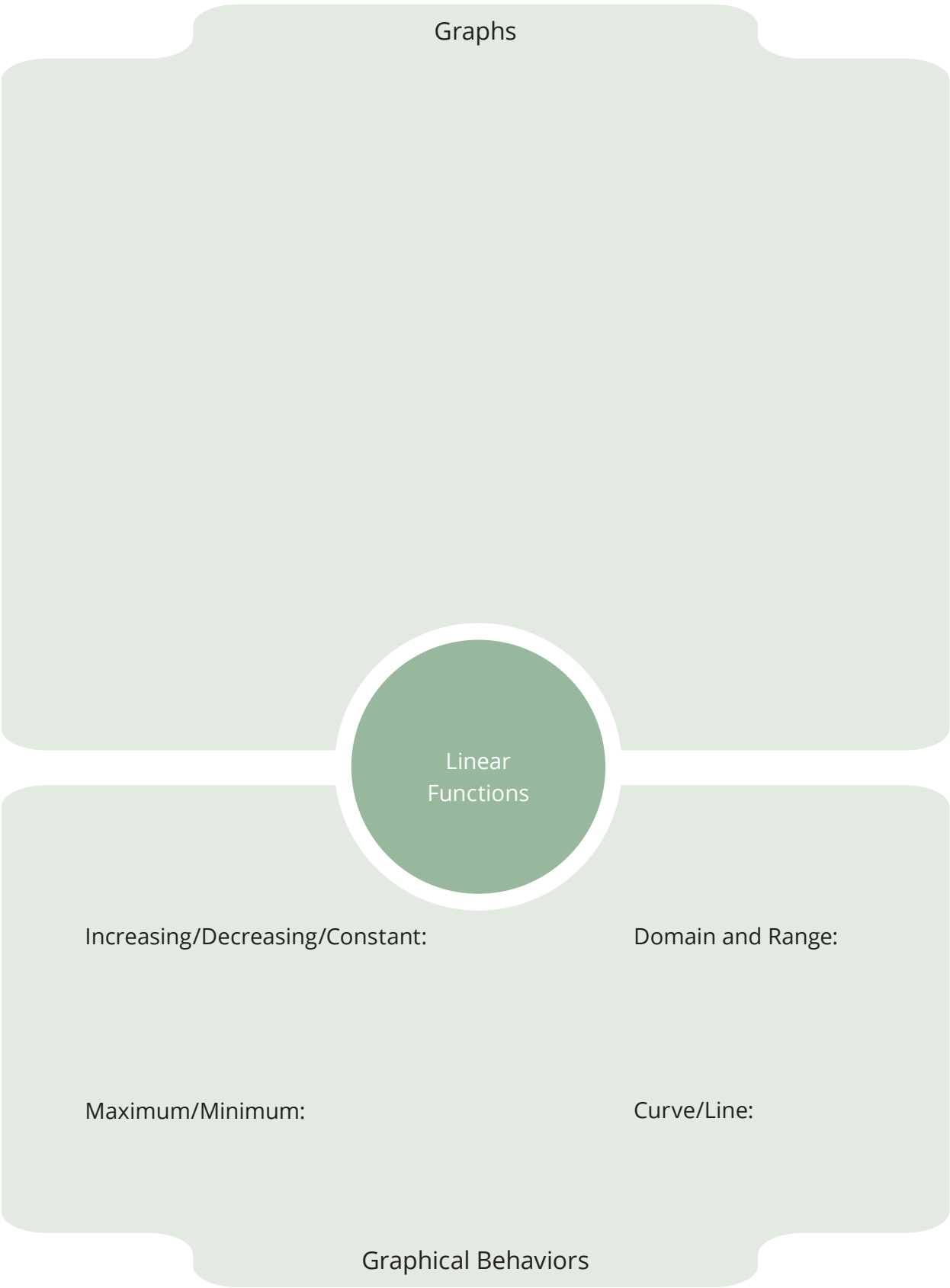
Interception!

Recall that the **x-intercept** is the point where a graph crosses the x-axis. The **y-intercept** is the point where a graph crosses the y-axis.

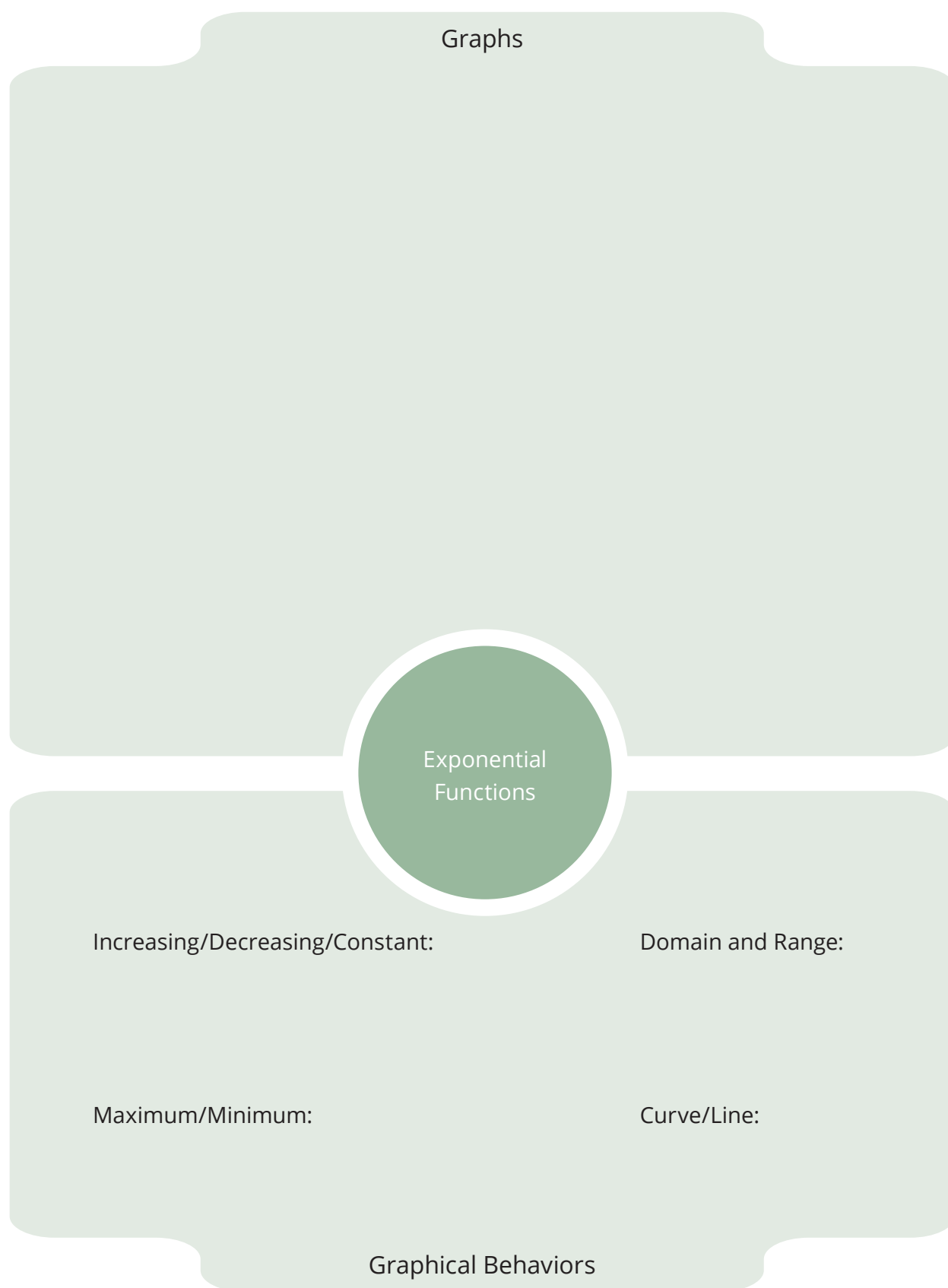
1. The graphs shown represent relations with just the x- and y-intercepts plotted. If possible, draw a function that has the given intercepts. If it is not possible, explain why not.



The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.



The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x + c$, where a , b , and c are real numbers, and b is greater than 0 but not equal to 1.



Assignment

Write

function notation
absolute maximum

increasing function
decreasing function

constant function
absolute minimum

Choose the term that best completes each statement.

- _____ is a way to represent equations algebraically that makes it more efficient to recognize the independent and dependent variables.
- When both the independent and dependent variables of a function increase across the entire domain, the function is called a(n) _____.
- A function has a(n) _____ if there is a point on its graph that has a y -coordinate that is greater than the y -coordinates of every other point on the graph.
- When the dependent variable of a function decreases as the independent variable increases across the entire domain, the function is called a(n) _____.
- If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a(n) _____.
- A function has a(n) _____ if there is a point on its graph that has a y -coordinate that is less than the y -coordinate of every other point on the graph.

Remember

A function is a relation that assigns to each element of the domain exactly one element of the range. The different function families include linear functions, exponential functions, quadratic functions, linear absolute value functions, and linear piecewise functions.

Practice

For each scenario, use graphing technology to determine the shape of its graph. Then identify the function family, whether it is increasing, decreasing, or a combination of both, has an absolute maximum or absolute minimum, and whether it is a smooth curve or straight line.

- A fitness company is selling DVDs for one of its new cardio routines. Each DVD will sell for \$15. Due to fixed and variable costs, the profit that the company will see after selling x DVDs can be represented by the function $P(x) = 11.5x - 0.1x^2 - 150$.
- The PARK SAFE commuter lot charges different rates depending on the number of hours a car is parked during the 5-day work week. The lot charges \$3 per hour for the first day, \$2 per hour for the next 2 days, and will charge \$1 per hour if the car is parked more than 3 days in the lot. The fees after x hours can be represented by the function shown.

$$f(x) = \begin{cases} 3x, & 0 \leq x \leq 24 \\ 72 + 2(x - 24), & 24 < x \leq 72 \\ x + 168 & 72 < x \leq 120 \end{cases}$$

3. Shari is going to put \$500 into an account with The People's Bank. The bank is offering a 3% interest rate compounded annually. The amount of money that Shari will have after x years can be represented by the function $A(x) = 500(1.03)^x$.
4. The Ace Calendar Company is going to buy a new 3D printer for \$20,000. In order to plan for the future, the owners are interested in the salvage value of the printer each year. The salvage value after x years can be represented by the function $S(x) = 20,000 - 2000x$.
5. An underwater camera has been placed in the center of the 25-meter pool at the Grandtown Aquatic Center to take pictures of swimmers during a swim meet. The camera will go off at different times depending on the distance of the swimmer to the camera. If the swimmer is moving at a constant rate of 1.28 meters per second, then the distance the swimmer is from the camera after x seconds can be represented by the function $d(x) = 1.28|x - 9.77|$.

Stretch

Graph both functions on the same screen using graphing technology. Use reasoning to classify the second function as a new family. Then describe the similarities and differences between the shapes of the graphs in terms of intervals of increase and decrease, maximums or minimums, and whether they are curves or lines.

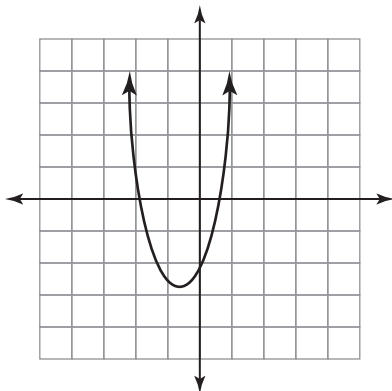
$$h(x) = x^2 + 9x + 14$$

$$p(x) = |x^2 + 9x| + 14$$

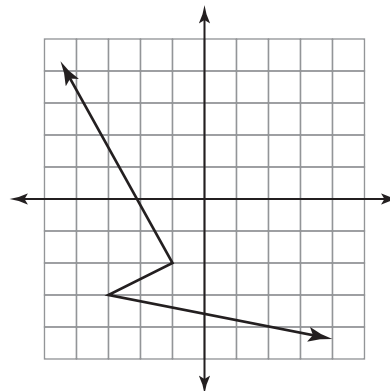
Review

1. Identify the axis of symmetry each graph has, if any, and identify the number of quadrants it passes through.

a.



b.



2. Solve the equation $2(9n - 6) + 52 = 2(7n + 8)$.
3. Evaluate the expression $\frac{3x^2 - 8(y + 2)}{2y}$ for $x = 8$ and $y = -2$.