

# Assignment

## Write

Write an example for each term using the dividend  $x^2 - 2x + 4$  and the divisor  $x + 1$ .

1. Factor Theorem
2. Polynomial long division
3. Remainder Theorem
4. Synthetic division

## Remember

A polynomial function  $p(x)$  has  $(x - r)$  as a factor if and only if the value of the function at  $r$  is 0, or  $p(r) = 0$ . When any polynomial equation or function  $f(x)$  is divided by a linear expression of the form  $(x - r)$ , the remainder is  $R = f(r)$  or the value of the equation or function when  $x = r$ .

## Practice

1. Use the Factor Theorem to determine whether each linear expression is a factor of the polynomial  $x^4 + x^3 - 17x^2 + 15x$ .
  - a.  $x + 3$
  - b.  $x + 5$
  - c.  $x - 1$
2. Factor each binomial over the set of real numbers.
  - a.  $4x^2 - 9y^2$
  - b.  $x^3 + 216$
3. The Polynomial Pool Company offers 10 different pool designs numbered 1 through 10. Each pool is in the shape of a rectangular prism. The volume of water in Pool Design  $x$  can be determined using the function  $V(x) = l(x) \cdot w(x) \cdot d(x) = 2x^3 + 18x^2 + 46x + 30$ , where  $l(x)$ ,  $w(x)$ , and  $d(x)$  represent the length, width, and depth of the pool in feet.
  - a. Determine the expressions for the functions  $w(x)$  and  $d(x)$  if  $l(x) = 2x + 2$  and the width of each pool is greater than the depth. Do not use a calculator.
  - b. Determine the length, width, and depth of Pool Design 9.
4. The function  $m(x) = 2x^2 + 6x - 7$  generates the same remainder when divided by  $(x - a)$  and  $(x - 2a)$  when  $a \neq 0$ . Calculate the value(s) of  $a$  and determine the corresponding factors.
5. The given table of values represents the function  $f(x) = x^3 + 9x^2 + 14x - 24$ .

$x$	-2	-1	0	1	2
$f(x)$	-24	-30	-24	0	48

- a. Determine one of the factors of  $f(x)$  without using a calculator. Explain your reasoning.
- b. Completely factor  $f(x)$  without using a calculator.
- c. Determine all of the zeros of  $f(x)$  without using a calculator.

## Stretch

The Rational Root Theorem states that if the polynomial

$P(n) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$  has any rational roots, then they must be of the form  $\pm \frac{p}{q}$ , where  $p$  is factors of  $a_0$  and  $q$  is factors of  $a_n$ .

1. Consider the function  $f(x) = 3x^3 - 4x^2 - 17x + 6$ .
  - a. Determine the values of  $a_0$  and  $a_n$  for this polynomial function.
  - b. Determine the values of  $p$ , or the factors of  $a_0$ .
  - c. Determine the values of  $q$ , or the factors of  $a_n$ .
  - d. Determine the possible rational zeros of the function.
  - e. Check all the possible rational zeros to determine whether any of them are roots of the function  $f(x)$ .

## Review

1. Determine the zeros of each function.
  - a.  $f(x) = 3(x - 1)^4 - 9$
  - b.  $f(x) = -2(x + 2)^3 - 1$
  - c.  $f(x) = 5(x + 3)^3$
2. Factor each polynomial over the set of real numbers.
  - a.  $16x^2 - 3x - 3$
  - b.  $x^4 - 10x^2 + 25$
3. Sketch the graph of  $f(x) = x^{17}$  and describe the end behavior of the graph.