# **Connecting the Dots**

Making Connections Between Arithmetic Sequences and Linear Functions

### Warm Up

Use what you know about arithmetic sequences to complete each task.

- 1. Write the first 5 terms of the sequence generated by  $a_n = 10 3(n 1)$ .
- 2. Given the function f(x) = -3x + 10, calculate f(1), f(2), f(3), f(4), and f(5).

# **Learning Goals**

- Use algebraic properties to prove the explicit formula for an arithmetic sequence is equivalent to the equation of a linear function.
- Relate the defining characteristics of an arithmetic sequence, the first term and common difference, and the defining characteristics of a linear function, the *y*-intercept and slope.
- Connect the slope of a line to the average rate of change of a function.

## **Key Terms**

- conjecture
- first differences
- average rate of change

You know that all sequences are functions. What type of function is an arithmetic sequence?

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# **Line Up in Sequential Order**

Kenyatta counts the number of new flowers that are blooming in her garden each day in the spring. Sequence A represents the number of new flowers on day 1, day 2, etc.

Sequence A: 3, 6, 12, 24, 48

She also measures the height of the first sunflower that has started blooming. Sequence B represents the height in centimeters of the sunflower on day 1, day 2, etc.

Sequence B: 3, 6, 9, 12, 15

1. For each sequence, determine whether it is arithmetic or geometric and write the explicit formula that generates the sequence. Then graph and label each on the coordinate plane.

**Sequence A:** 

**Sequence B:** 

2. List at least three common characteristics of the graphs. How do the two sequences compare?

# **Connecting Forms**



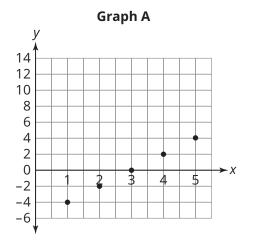
Consider the four explicit formulas, each representing a different arithmetic sequence.

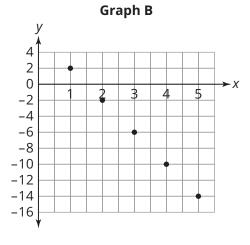
- $a_n = 2 4(n 1)$   $a_n = -4 + 2(n 1)$   $a_n = 4 + 2(n 1)$

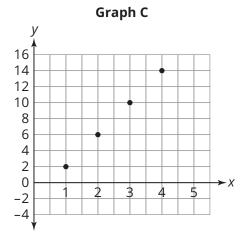
1. Match each explicit formula with its graph. Describe the strategies you used.

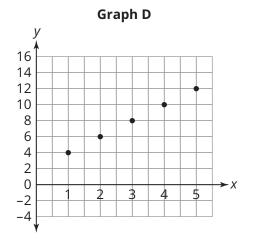


How do you know by the form of the explicit formula that it represents an arithmetic sequence?









2. Consider the set of graphs and identify the function family represented. Based on these formulas and graphs, do you think that all arithmetic sequences belong to this function family? Explain your conjecture.

A conjecture is a mathematical statement that appears to be true, but has not been formally proven.

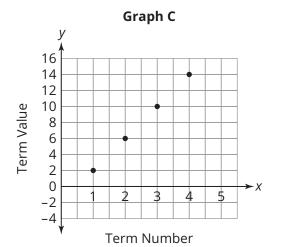




An arithmetic sequence is a sequence of numbers in which the differences between any two consecutive terms is constant. The explicit formula is of the form  $a_n = a_1 + d(n-1)$ .

Let's take a closer look at the relationship between arithmetic sequences and the family of linear functions. You know a lot about each relationship.

The explicit formula and the table of values that represents Graph C are shown.



$$a_n = 2 + 4(n-1)$$

Term Number <i>n</i>	Tei Val	
1	<i>a</i> <sub>1</sub>	2
2	$a_2$	6
3	<i>a</i> <sub>3</sub>	10
4	<i>a</i> <sub>4</sub>	14

- 3. Describe the domain of the sequence.
- 4. Identify the common difference in each representation.
- 5. Draw a line to model the linear relationship seen in the graph. Then write the equation to represent your line. Describe your strategy.
- 6. Describe the domain of the graph of the linear model.

An explicit formula is one equation that you can use to model an arithmetic sequence. You can rewrite the explicit formula for the arithmetic sequence  $a_n = 2 + 4(n - 1)$  in function notation.

A linear function written in general form is f(x) = ax + b, where a and b are real numbers. In this form a represents the slope and b represents the y-intercept.

#### Worked Example

You can represent  $a_n$  using function notation.

$$a_n = 2 + 4(n - 1)$$
  
 $f(n) = 2 + 4(n - 1)$ 

Next, rewrite the expression 2 + 4(n - 1).

$$f(n) = 2 + 4n - 4$$
 Distributive Property  
=  $4n + 2 - 4$  Commutative Property  
=  $4n - 2$  Combine Like Terms

So,  $a_n = 2 + 4(n - 1)$  written in function notation is f(n) = 4n - 2.

- 7. Compare the equation you wrote to model the graph of the arithmetic sequence to the explicit formula written in function form. What do you notice?
- 8. Compare the common difference with the slope. What do you notice?
- 9. Explain why Hank's reasoning is not correct.

#### Hank

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The y-intercept of a linear function is the same as the first term of an arithmetic sequence.

# Connecting Constant Difference and Slope





At the end of the last show, stagehands at the community theater stack the audience chairs and place them in storage. The height of one chair is 34 inches, and as each additional chair is stacked, the height increases by 8 inches.

- 1. Write an equation using the explicit formula to represent this scenario.
- 2. Rewrite the explicit formula in function form and define your variables.
- 3. Compare the two algebraic representations of this scenario.
  - a. How is the value of d in the explicit formula related to the value of a in function form? How are d and a represented in the scenario? Be sure to include units of measure.
  - b. How is the value of  $a_1$  from the explicit formula related to b in function form? How are  $a_1$  and b represented in the scenario?
  - c. If  $a_1$  represents the first term of a sequence, what does  $a_0$  represent? How can you rewrite the arithmetic sequence using  $a_0$ ?

$$a_n = \underline{\hspace{1cm}}$$

In a sequence, the common, or constant, difference is the difference in term values between

consecutive terms.

In a linear function, the slope describes the direction and steepness of the line. It is the difference in output values divided by the difference in corresponding input values.

Remember:

The slope formula is  $\frac{y_2 - y_1}{x_2 - x_1}$ .

When you see a graph or rewrite an explicit formula for an arithmetic sequence, it is apparent that it represents a linear function. However, the structure of a table requires other strategies to determine whether it represents a linear function.

One strategy is to examine *first differences*. **First differences** are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear.

The tables that represent the explicit formula and function form of the stacking chairs scenario are shown.

n	a <sub>n</sub>
1	34
2	42
3	50
4	58

X	y = f(x)
0	26
1	34
2	42
3	50

# 4. Determine the first differences in each table to verify they both represent a linear relationship.

The expression y = f(x) means that the value of y depends on the value of x. That is, for different values of x, there is a function f which determines the value of y.



5. Parks and Eva Cate agree that the table shown represents a linear relationship because there is a constant difference between consecutive points. Parks claims the slope is 5 and Eva Cate claims the slope is −5.

Who's correct? Explain your reasoning.



What does slope describe?

X	у	
1	22	22 - 17 = 9
2	17	<u> </u>
3	12	17 - 12 = 5
4	7	12 - 7 = 5

6. Use first differences to determine whether each table represents a linear function. Then, create your own table to represent a linear function. Describe your strategy.

a.

Х	у
5	12
6	15
7	21
8	30

b.

Х	у
-2	18
-1	14
0	10
1	6

C.

Х	У
10	1
11	4
12	9
13	16

d.

Х	у

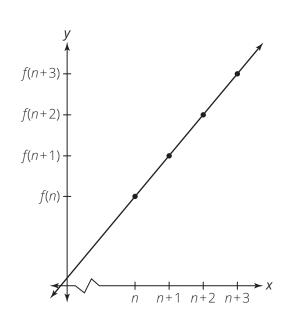


In the previous activity, you determined that the constant difference of the chair heights and the slope of the line are both equal to 8 inches per chair. Is the slope of a linear function always equal to the constant difference of the corresponding arithmetic sequence?

Consider the graph of the arithmetic sequence represented by the general linear function f(x) = ax + b.

- 1. Identify the constant difference of the sequence in the graph.
- 2. Complete the table for consecutive values of the input.

Term Number	Ter	m Value
n	f(n)	a(n) + b
n + 1	f(n+1)	a(n + 1) + b
n + 2		
n + 3		



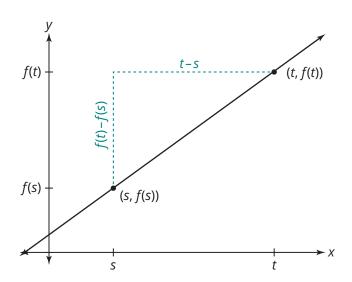
3. Select any two consecutive input values in the sequence.
Use the expressions for the term values to determine the constant difference of the sequence.

# NOTES

Recall that the slope of a line is constant between any two points on the line, not just consecutive points.

- 4. Use the table and graph from Question 2 to complete each task.
  - a. Identify the slope of the function on the graph.
  - b. Select two non-consecutive points in the table and determine the slope between those two points.

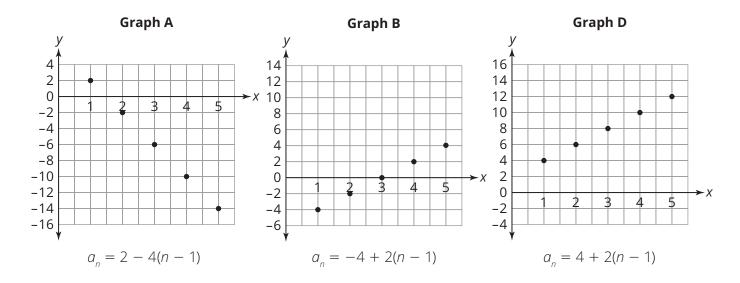
The slope, a, is equal to the constant difference. Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is  $\frac{f(t)-f(s)}{t-s}$ . This represents the change in the output as the input changes from s to t.



5. Show that the slope formula and the average rate of change formula represent the same ratio.



The remaining explicit formulas and graphs from Activity 1 are shown.



- 1. For each graph and arithmetic sequence, complete each task.
  - a. Identify the constant difference in the explicit formula and on the graph.
  - b. Rewrite each explicit formula in function notation.
  - c. Verify that the constant difference is the same as the slope of the linear function.
  - d. Describe how the first term in the explicit formula is related to the *y*-intercept of the function.

# Describing Constant Sequences

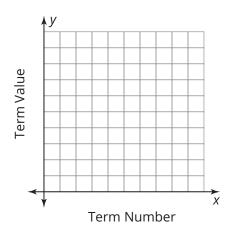


Kenyatta's cat loves to knock over flowerpots. Each morning, she counts the number of flowerpots her cat knocked over during the night before she uprights them again. Sequence C represents the number of flowerpots knocked over on day 1, day, 2, etc.

Sequence C: 3, 3, 3, 3, 3

1. Determine the constant difference and write the explicit formula to represent Sequence C. Then create a table of values and graph it.

Term Number (n)	Term Value (a <sub>n</sub> )



- 2. Use function notation to write an equation representing the relationship between the number of days, *x*, and the number of new flowerpots, *C*. Interpret the function in terms of the context.
- 3. Prove that the slope of C(x) is equal to the common difference of Sequence C.

4. Write the function, D(x), to model this sequence.

5. Prove D(x) is a constant function.



If the values of the dependent variable of a function remain constant over the entire domain, then the function is called a constant function.

# TALK the TALK



# **Making It Plain and Clear**

You have proven that all arithmetic sequences can be represented by linear functions.

1. Complete the graphic organizer to summarize the connections between arithmetic sequences and linear functions. Then describe how you can tell a linear relationship exists given a table of values or a graph.

#### **Equation**

Arithmetic Sequence	Linear Function	Mathematical
$a_n = a_1 + d(n-1)$	f(x) = ax + b	Meaning
$Q_n$		
d		
n		
$a_1 - d$		

CHARACTERISTICS **FUNCTIONS** 

**Table of Values** 

Graph

#### Write

Describe how the terms constant difference, slope, and average rate of change are related.

#### Remember

The explicit formula of an arithmetic sequence can be rewritten as a linear function in the general form f(x) = ax + b, where a and b are real numbers, using algebraic properties. The constant difference of an arithmetic sequence is always equal to the slope of the corresponding linear function.

#### **Practice**

- 1. Rakesha claims that the equation f(n) = 5n 7 is the function notation for the sequence that is represented by the explicit formula  $a_n = -2 + 5(n 1)$ . James doesn't understand how this can be the case.
  - a. Help James by listing the steps to write the explicit formula of the given sequence in function notation. Provide a rationale for each step.
  - b. Graph the function. Label the first 5 values of the sequence on the graph.
- 2. Determine whether each table of values represents a linear function. For those that represent linear functions, write the function. For those that do not, explain why not.

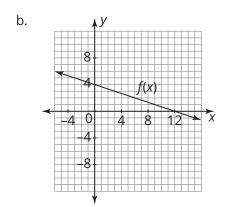
a.	х	f(x)
	3	14
	4	18
	5	23
	6	29

•	X	f(x)		
	0	2		
	1	-1		
	2	-4		
	3	-7		

X	f(x)		
1	11		
2	16		
3	21		
4	26		

3. Calculate the average rate of change for each linear function using the formula. Show your work.

_		
a.	х	f(x)
	3	-4
	7	4
	9	8
	12	14



c.

#### Stretch

Craig left his house at noon and drove 50 miles per hour until 3 PM. Then he drove the next 5 hours at 70 miles per hour. Graph Craig's driving trip and calculate the average rate of change for the entire trip.

#### Review

Evaluate each function for the given values.

1. 
$$f(x) = 3x - 10$$

2. 
$$f(x) = 6$$

3. 
$$f(x) = 9x + 7 - 3x$$

b. 
$$f(-2)$$

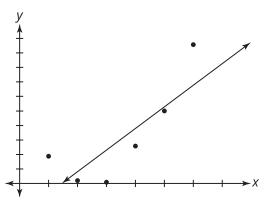
4. The linear regression equation for the given data is y = -x + 19.7. Complete the table for the linear regression equation, rounding your answers to the nearest tenth. Then construct and interpret a residual plot.

Х	У	Predicted Value	Residual Value
2	17		
4	16		
6	15		
8	12		
10	9		
12	8		

5. The linear regression equation for the given data is y = 3.93x - 11.33, r = 0.8241. Consider the scatterplot, the correlation coefficient, and the corresponding residual plot. State whether a linear model is appropriate for the data.

Х	2	4	6	8	10	12
У	9	2	1	12	25	48

Scatter Plot and Line of Best Fit



Residual Plot

