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It's Literally About Literal Equations

Literal Equations

Warm Up

The formula for the circumference of a circle is $C = 2\pi r$. Determine the radius for each circle with the given circumference. Use 3.14 for π and round to the nearest tenth of a unit, if necessary.

1. $C = 62.8$ in.

2. $C = 10$ cm

3. $C = 15.7$ ft

4. $C = 48$ mm

Learning Goals

- Rewrite linear equations in different forms.
- Analyze the structure of different forms of linear equations.
- Recognize and use literal equations.
- Rearrange literal equations to highlight quantities of interest.

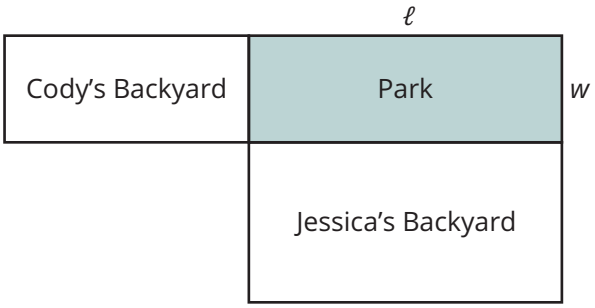
Key Term

- literal equation

You have used different properties to solve linear equations to determine which value makes the equation true. How can you use those same properties to solve for one specific variable in an equation that has multiple variables?

Perimeter Perspectives

Cody and Jessica have rectangular backyards that each share a side with a neighborhood park. The sides shared with the park are adjacent to each other as shown.



Cody and Jessica are each responsible for constructing their own fence to separate their yard from the park. The city gives them the measure of the perimeter of the park in feet. So that each knows how much fencing to buy, Cody needs to determine the width of the park, and Jessica needs to determine the length of the park.

Remember:

The formula for the perimeter of a rectangle is $P = 2l + 2w$.

1. **How can you write an equation for the perimeter of the park from each person's perspective to help them determine the information they need?**

2. **Show that the two equations are equivalent.**

ACTIVITY
2.1 Equations in Different Forms

The slope, *x*-intercept, and *y*-intercept are each important characteristics of linear functions. The structure of equations can reveal these characteristics of a function.

1. Consider the equation $y = -\frac{2}{3}x + 4$. Determine each characteristic and then explain your work.

a. slope

b. *y*-intercept

c. *x*-intercept

d. How did you determine each characteristic?

2. Consider the equation $y = 4(x - 2)$. Determine each characteristic and then explain your work.

a. slope

b. *y*-intercept

c. *x*-intercept

d. How did you determine each characteristic?

3. Consider the equation $3x - 2y = 8$. Determine each characteristic and then explain your work.

a. slope

b. *y*-intercept

c. *x*-intercept

d. How did you determine each characteristic?

There are three useful forms of linear equations.

General Form	Factored Form	Standard Form
$y = ax + b$	$y = a(x - c)$	$Ax + By = C$

In general and factored form, the values of a, b, and c can be any real numbers. In standard form, however, there are constraints on the variables: A must be a positive integer, and A and B cannot both be 0.

4. Identify the form for each equation in Questions 1 through 3.

Consider how the structure of the standard form of a linear function reveals its key characteristics.

5. For the equation $Ax + By = C$, determine the slope, x-intercept, and y-intercept.

6. Which form of a linear equation is more efficient for determining each characteristic?

a. slope

b. x-intercept

c. y-intercept

7. If you want to graph an equation using your calculator, which form is more efficient? Explain your reasoning.

ACTIVITY
2.2

Rewriting Literal Equations

Literal equations are equations in which the variables represent specific measures. You most often see literal equations when you study formulas. These literal equations can be manipulated in order to allow you to solve for one specific variable.

You have used a common literal equation, the formula for converting degrees Fahrenheit to degrees Celsius.

$$C = \frac{5}{9}(F - 32)$$

Josh is talking on the phone to his friend Greg who lives in Europe. Greg is used to describing temperatures in °C, while Josh is used to describing temperatures in °F. Greg can use the formula above to quickly convert the temperatures Josh describes to °C.

1. **How can you rewrite the formula so that Josh can quickly convert the temperatures that Greg describes to °F? Justify your solution method.**

2. **Josh tells Greg that the temperature where he lives is currently 77°F. Greg tells Josh that the temperature where he lives is currently 30°C. Josh says it is warmer where he lives and Greg says it is warmer where he lives. Who is correct? Explain your reasoning.**

3. Maya, Sherry, and Brian each convert the given formula to degrees Fahrenheit.

a. Explain Maya's reasoning.

Maya

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$\frac{9C}{5} + \frac{160}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 32 = F$$

Sherry

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - 32$$
$$9(C) = 9\left(\frac{5}{9}F - 32\right)$$
$$9C = 5F - 288$$
$$9C + 288 = 5F$$
$$\frac{9C}{5} + \frac{288}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 57.6 = F$$

b. Explain the error in Sherry's work.

Brian

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 45F - 1440$$
$$9C + 1440 = 45F$$
$$\frac{9C}{45} + \frac{1440}{45} = \frac{45F}{45}$$
$$\frac{1}{5}C + 32 = F$$

c. Explain the error in Brian's work.

4. **Carlos and Mikala do not like working with fractions. Each rewrites the equation so that it does not have fractions. Their work is shown.**

Carlos

$$F = \frac{9}{5}C + 32$$
$$(5)F = 5\left(\frac{9}{5}C + 32\right)$$
$$5F = 9C + 160$$
$$5F - 9C = 160$$

Mikala

$$C = \frac{5}{9}(F - 32)$$
$$(9)C = (9)\left(\frac{5}{9}(F - 32)\right)$$
$$9C = 5(F - 32)$$
$$9C = 5F - 160$$
$$9C - 5F = -160$$

Carlos and Mikala got two different equations. Who is correct? Explain your reasoning.

5. **In the original equations, the coefficients $\frac{9}{5}$ and $\frac{5}{9}$ as well as the constant 32 had meaning based on temperature. What do the coefficients, 9 and 5, and the constant, 160, represent in Carlos's and Mikala's equations?**

6. **How is the literal equation $y = a(bx + c)$ similar to the equation for converting °F to °C? How would you solve this equation for the variable x?**

ACTIVITY
2.3
Solving Literal Equations for Specific Variables

Convert each literal equation to solve for the given variable.

Remember:

Volume is measured in cubic units since it calculated using three dimensions. Height measures only one dimension.

1. **Think Inside the Box is manufacturing new boxes for You Pack 'Em, We Ship 'Em (YPEWSE). YPEWSE told Think Inside the Box that the boxes must have a specific volume and area of the base. However, YPEWSE did not specify a height for the boxes.**

   a. **Write a literal equation to calculate the volume of a box. Then convert the volume formula to solve for height.**

   b. **YPEWSE specified the volume of the box must be 450 cubic inches and the area of the base must be 75 square inches. Use your formula to determine the height of the new boxes.**

The formula for the volume of a cone is $V = (\pi)r^2\left(\frac{h}{3}\right)$ where r is the radius and h is the height.

2. **The volume of an ice cream cone is the measure of how much ice cream fits inside the cone. An ice cream cone company wants to make an ice cream cone with a greater height that still holds the same amount of ice cream.**

   a. **Write an equation to calculate the volume of a cone. Then convert the equation to solve for the height.**

   b. **Explain how your equation determines a linear measurement when the original equation determined a cubic measurement.**

3. **The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is its height and b_1 and b_2 are the lengths of each base.**

   a. **Convert the area formula to solve for the height.**

   b. **Use your formula to determine the height of a trapezoid with an area of 24 cubic centimeters and base lengths of 9 cm and 7 cm.**

4. **For the given literal equation $Z = \frac{A}{B} + \frac{C}{D}$, solve for each variable given. Justify your solution method.**

   a. **A**

   b. **D**

NOTES

TALK the TALK

This Ought to Be Duck Soup

1. **The formula for the volume of a cylinder is $V = 2\pi rh$ where V is the volume, r is the length of the radius of the base, and h is the height. Convert the formula to solve for the height.**

2. **The volume of a can of soup is 37.68 cubic inches and the length of the radius of the base of the can is 1.5 inches. Use the formula to determine the height of the can of soup. Use 3.14 for π.**

3. **The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$ where SA is the surface area, r is the length of the radius of the base, and h is the height. Convert the formula to solve for the height.**

4. **The surface area of a can of soup is 51.81 square inches and the length of the radius of the base is 1.5 inches. Use your formula to determine the height of the can of soup. Use 3.14 for π.**

Assignment

Write

Define the term *literal equation* in your own words.

Remember

Literal equations can be rewritten using properties of equality to allow you to solve for one specific variable.

Practice

1. In the USA, the shoe sizes for men are approximated by the equation $3f - s = 24$, where f represents the length of the foot in inches and s represents the shoe size.
   a. The average man's foot is 11.5 inches long. What is the average man's shoe size?
   b. Use the function to determine the x- and y-intercept. State the meaning of each in terms of this problem situation.
   c. Which form can most easily be used to determine the slope of this equation? Determine the slope of this equation and describe what it means in terms of the problem situation.
2. The boxes that shoes come in are often used in other capacities once the shopper has bought the shoes. Sometimes the boxes are used to hold other items, so it is helpful to know the volume of the box.
   a. Write the equation to solve for the volume of the shoe box.
   b. If the area of the base of the box is 112 square inches and the height is 3.5 inches, what is the volume of the box?
   c. Rewrite the equation to solve for width. Show your work.
   d. A box has a volume of 456 cubic inches, with a length of 1 foot and a height of 4 inches. Determine the width of the box.
3. Solve each equation for the specified variable.
   a. $V = \frac{1}{3}Bh$ for B
   b. $I = prt$ for r
   c. $\frac{x + y}{3} = 6$ for y
   d. $A + B + C = 180$ for C

Stretch

A simple pendulum is made of a long string and a small metal sphere. The period of oscillation can be found by the formula $T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$, where g is the acceleration due to gravity, and L is the length of the string. Solve the formula for g, the acceleration due to gravity.

Review

1. The Peters Creek restaurant has an all-you-can eat shrimp deal. Currently, the cost of the deal is 50 cents per shrimp, with free soft drinks. The cost for the shrimp deal is modeled by the function $c(x) = 0.50x$, where x represents the number of shrimp eaten. A new manager decides to change the cost of the deal to 25 cents per shrimp, but a \$5.00 charge for soft drinks. Let $p(x)$ be the function that represents the new cost for the all-you-can-eat shrimp deal.
   a. Sketch the graph of $c(x)$ and $p(x)$ on the same coordinate plane.
   b. Complete the table of corresponding points on $p(x)$.
   c. Write an equation for $p(x)$ in terms of $c(x)$. Describe the transformation performed on $c(x)$ to produce $p(x)$.
   d. Write the equation for the function $p(x)$ in general form.

x	$p(x)$
0	
10	
40	
60	
70	
90	

2. Solve the equation. Write the properties that justify each step.
$$-\frac{2}{3}x(-9x + 24) = 2x - 4$$
3. Determine if the equation has one solution, no solution, or infinite solutions.
$$3\left(2 + \frac{2}{3}x\right) = 5 + 2(x + 1)$$
4. The table shows the relationship between y and x. Write an equation that represents the relationship between the variables.

x	y
−3	2
−1	6
1	10
3	14
6	20

5. A clothing store decides to give out bonus points that can be used for future purchases. If a customer applies for a bonus card they are automatically given 50 bonus points. After that, they get 25 bonus points for every \$1.00 that they spend. Write an equation that shows the number of bonus points, b, that a customer will earn for x dollars that they spend.