1

Double the Fun

Introduction to Systems of Equations

Warm Up

Determine an ordered pair that represents a solution to each equation.

1. $4x + 7y = 24$

2. $5x - 2y = -6$

3. $\frac{1}{2}x + \frac{3}{4}y = 10$

Learning Goals

- Write equations in standard form.
- Determine the intercepts of an equation in standard form.
- Use intercepts to graph an equation.
- Write a system of equations to represent a problem context.
- Solve systems of linear equations exactly and approximately.
- Interpret the solution to a system of equations in terms of a problem situation.
- Use slope and *y*-intercept to determine whether the system of two linear equations has one solution, no solution, or infinite solutions.

Key Terms

- system of linear equations
- inconsistent systems
- consistent systems

You have examined different linear functions and solved for unknown values. How can you solve problems that require two linear functions? How many solutions exist when you consider two functions at the same time?

GETTING STARTED

Ticket Tabulation

The Marshall High School Athletic Association sells tickets for the weekly football games. Students pay $5 and adults pay $10 for a ticket. The athletic association needs to raise $3000 selling tickets to send the team to an out-of-town tournament.

1. Write an equation to represent this situation.

2. What combination of student and adult ticket sales would achieve the athletic association's goal?

3. Compare your combination of ticket sales with your classmates'. Did you all get the same answer? Explain why or why not.

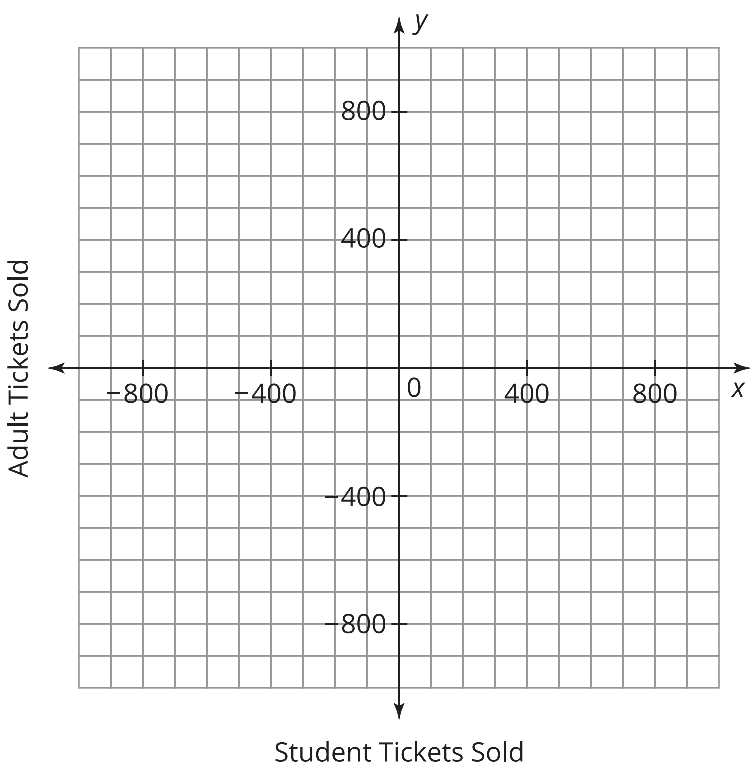
ACTIVITY
1.1

Analyzing the Graph of an Equation in Standard Form

Consider the goal of the athletic association described in the previous activity. Let s represent the number of student tickets sold, and let a, represent the number of adult tickets sold. Written in standard form, the equation that represents the situation is $5s + 10a = 3000$.

One efficient way to graph a linear function in standard form is to use x- and y-intercepts. You can calculate the x-intercept by substituting $y = 0$ and solving for x. You can calculate the y-intercept by substituting $x = 0$ and solving for y.

1. Use the *x*-intercept and *y*-intercept to graph the equation.



2. Determine the domain and range of each.

a. the mathematical function

b. the function modeling the real-world situation

Remember:

The standard form of a linear equation is $Ax + By = C$ where A, B, and C are constants and A and B are not both zero.

Ask yourself:

Which quantity is represented on each axis?

3. Explain what each intercept means in terms of the problem situation.

4. Identify the slope of the function. Interpret its meaning in terms of the problem situation.

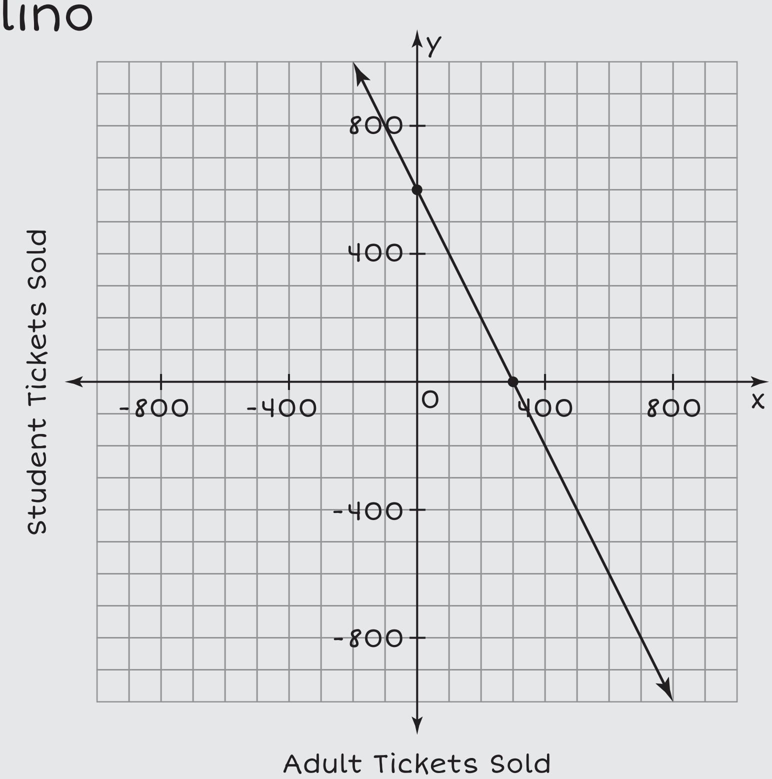
Ask yourself:

What does each point on the graph of an equation represent?

5. How can you use the graph to determine a combination of ticket sales to meet the goal of $3000?

6. Felino graphed the equation $5s + 10a = 3000$ in a different way. Explain why Felino's graph is correct.

Felino



7. **Use Felino's graph to describe the domain and range of each.**

   a. **the mathematical function**

   b. **the function modeling the real-world situation**

8. **Explain what each intercept means in terms of the problem situation.**

9. **Identify the slope of the function. Interpret its meaning in terms of the problem situation.**

10. **Compare the domain and range of the two functions that model the real-world situation. What do you notice?**

11. **Compare the *x*-intercepts and the *y*-intercepts of the two graphs. What do you notice?**

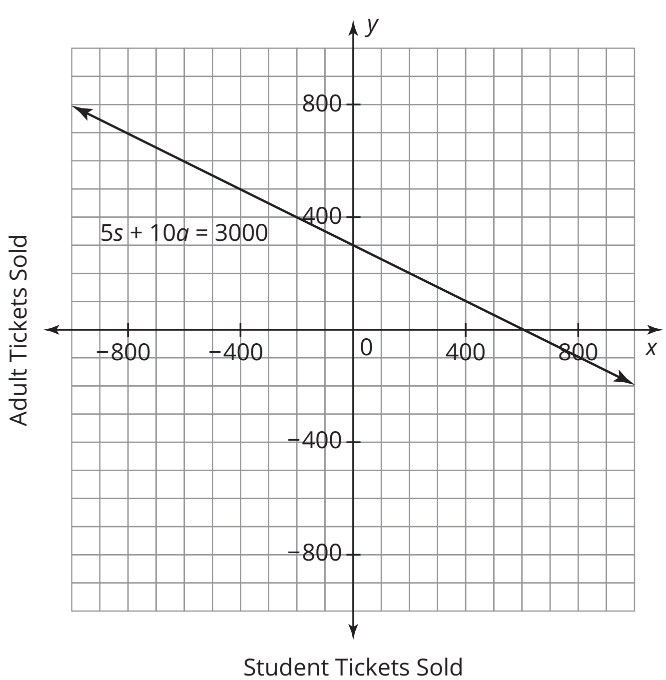
12. **Is there a way to determine the total amount of money collected from either graph? Explain why or why not.**

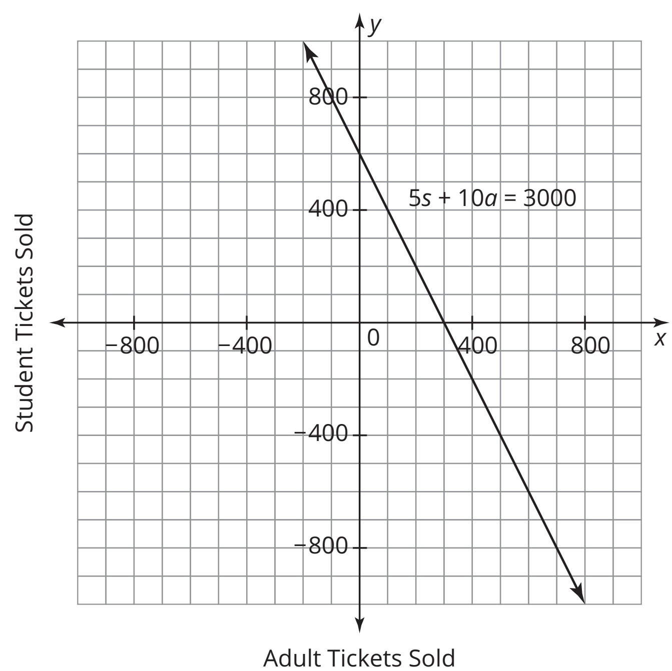
ACTIVITY 1.2 Determining the Solution to a System of Linear Equations

The athletic director of the Marshall High School Athletic Association says that 450 total tickets were sold to the home game.

1. Write an equation that represents this situation. Let *s* represent the number of student tickets sold, and let *a* represent the number of adult tickets sold.

The coordinate planes shown already contain the function that models the earnings from ticket sales.





2. Use *x*- and *y*-intercepts to graph the function modeling the total number of tickets sold on each coordinate plane.

3. **If the athletic association reached its goal of $3000 in ticket sales, how many of each type of ticket was sold? Is there more than one solution?**

Remember:

According to the situation, 450 tickets were sold to the game.

4. **Use technology to locate the exact point of intersection. Explain the process you used.**

5. **Justify algebraically that your solution is correct.**

The two graphed equations share a relationship between quantities. Each equation describes a relationship between the number of adult tickets sold and the number of student tickets sold. In one, the relationship is defined by the cost of each ticket and the total amount collected. In the second, the relationship is defined by the total number of tickets sold. The two equations together form a *system of linear equations*. When two or more linear equations define a relationship between quantities, they form a **system of linear equations**.

ACTIVITY

1.3

Systems with No Solution, One Solution, or an Infinite Number of Solutions

Marcus and Phillip are in the Robotics Club. They are both saving money to buy materials to build a new robot.

Marcus opens a new bank account. He deposits $25 that he won at a robotics competition. He also plans on depositing $10 a week that he earns from tutoring. Phillip decides he wants to save money as well. He already has $40 saved from mowing lawns over the summer. He plans to also save $10 a week from his allowance.

1. Write equations to represent the amount of money Marcus saves and the amount of money Phillip saves.

2. Use your equations to predict when Marcus and Phillip will have the same amount of money saved.

You can prove your prediction by solving and graphing a system of linear equations.

3. Analyze the equations in your system.

a. How do the slopes compare? Describe what this means in terms of this problem situation.

b. How do the *y*-intercepts compare? Describe what this means in terms of this problem situation.

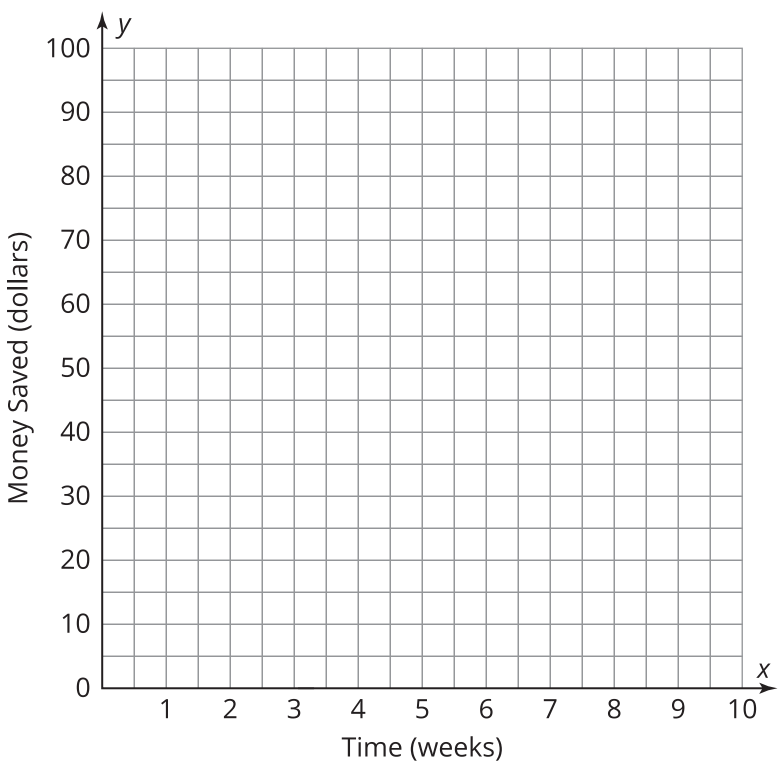
4. **Determine the solution of the system of linear equations algebraically and graphically.**

   a. **Use the substitution method to determine the intersection point.**

   b. **Does your solution make sense? Describe what this means in terms of the problem situation.**

   c. **Predict what the graph of this system will look like. Explain your reasoning.**

   d. **Graph both equations on the coordinate plane.**



5. **Analyze the graph you created.**

   a. **Describe the relationship between the graphs.**

   b. **Does this linear system have a solution? Explain your reasoning.**

6. **Was your prediction in Question 2 correct? Explain how you algebraically and graphically proved your prediction.**

Tonya is also in the Robotics Club and has heard about Marcus's and Phillip's savings plans. She wants to be able to buy her new materials before Phillip, so she opens her own bank account. She is able to deposit $40 in her account that she has saved from her job as a waitress. Each week she also deposits $4 from her tips.

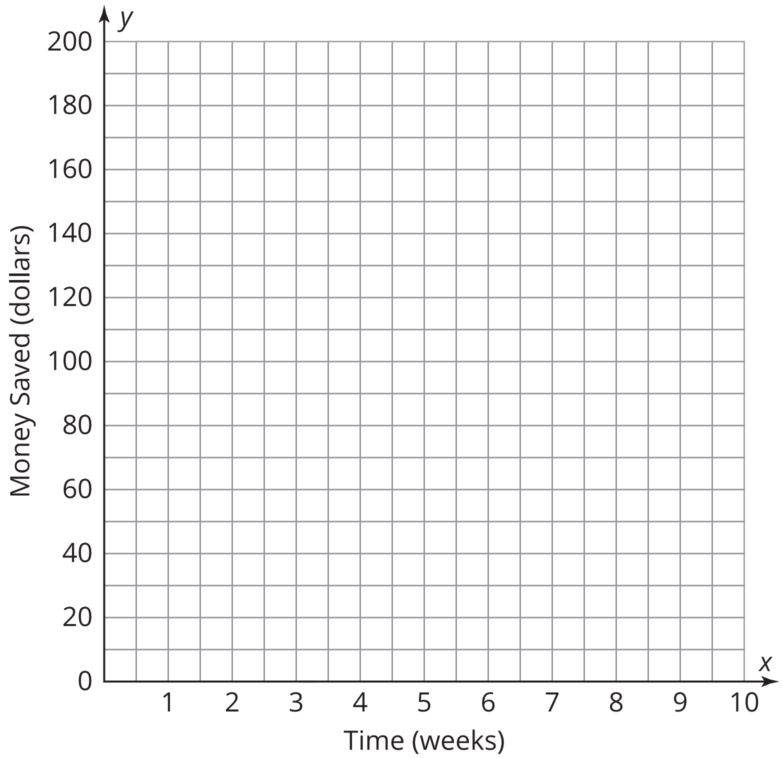
7. **Use equations and graphs to determine when Tonya and Phillip have saved the same amount of money.**

   a. **Write a linear system to represent the total amount of money Tonya and Phillip have saved after a certain amount of time.**

Remember:

Don't forget to define your variables!

b. Graph the linear system on the coordinate plane.



c. Do the graphs intersect? If so, describe the meaning in terms of this problem situation.

Phillip and Tonya went on a shopping spree this weekend and spent all their savings except for $40 each. Phillip is still saving $10 a week from his allowance. Tonya now deposits her tips twice a week. On Tuesdays she deposits $4 and on Saturdays she deposits $6.

8. Phillip claims he is still saving more each week than Tonya.

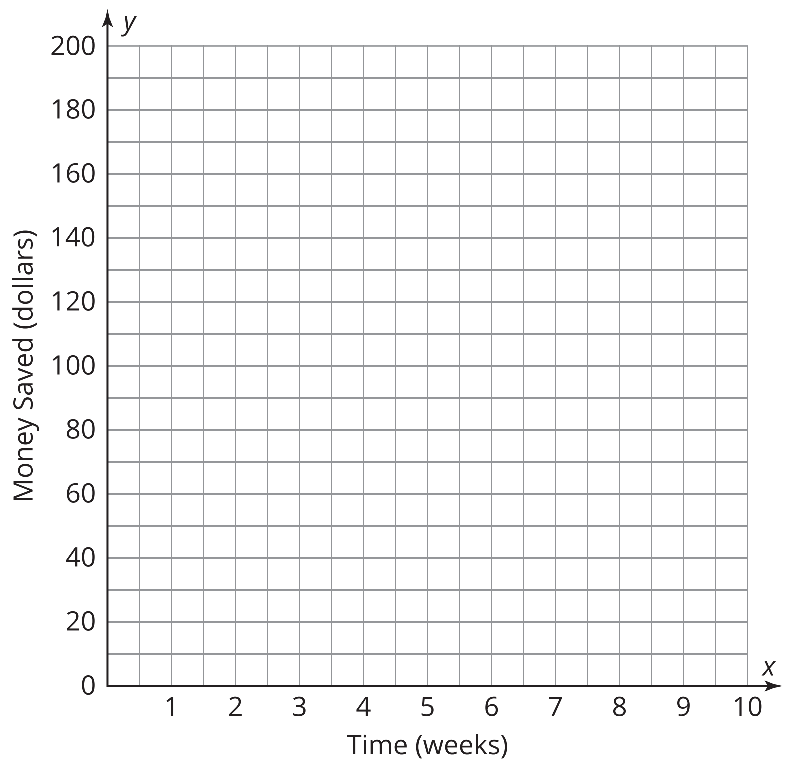
a. Do you think Phillip's claim is true? Explain your reasoning.

b. How can you prove your prediction?

9. Prove your prediction algebraically and graphically.

a. Write a new linear system to represent the total amount of money each friend has after a certain amount of time.

b. Graph the linear system on the coordinate plane.



10. Analyze the graph.

a. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

b. Algebraically prove the relationship you stated in part (a).

c. How does this solution prove the relationship? Explain your reasoning.

11. Was Phillip's claim that he is still saving more than Tonya a true statement? Explain why or why not.

NOTES

TALK the TALK

Beating the System

1. How does solving a linear system in two variables compare to solving an equation in one variable?

A system of equations may have one unique solution, no solution, or infinite solutions. Systems that have one or many solutions are called **consistent systems.** Systems with no solution are called **inconsistent systems.**

2. Complete the table.

System of Two Linear Equations	Consistent Systems		Inconsistent Systems
Description of *y*-Intercepts	*y*-intercepts can be the same or different	*y*-intercepts are the same	*y*-intercepts are different
Number of Solutions			
Description of Graph			

3. Explain why the *x*- and *y*-coordinates of the points where the graphs of a system intersect are solutions.

Assignment

Write

Define each term in your own words.

1. consistent systems
2. inconsistent systems

Remember

When two or more equations define a relationship between quantities, they form a system of linear equations. The point of intersection of a graphed system of linear equations is the solution to both equations. A system of linear equations can have one solution, no solution, or infinite solutions.

Practice

1. Mr. Johanssen gives his class 50-question multiple choice tests. Each correct answer is worth 2 points, while a half point is deducted for each incorrect answer. If the student does not answer a question, that question does not get any points.
   a. A student needs to earn 80 points on the test in order to keep an A grade for the semester. Write an equation in standard form that represents the situation. Identify 3 combinations of correct and incorrect answers that satisfy the equation.
   b. Determine the *x*- and *y*-intercepts of the equation and use them to graph the equation. Explain what each intercept means in terms of the problem situation.
2. Wesley owns a dairy farm. In the morning, it takes him 0.3 hour to set up for milking the cows. Once he has set up, it takes Wesley 0.2 hour to milk each cow by hand. He is contemplating purchasing a milking machine in hopes that it will speed up the milking process. The milking machine he is considering will take 0.4 hour to set up each morning and takes 0.05 hour to milk each cow.
   a. Write a system of linear equations that represents the total amount of time Wesley will spend milking the cows using the two different methods.
   b. Graph both equations on a coordinate plane.
   c. Estimate the point of intersection. Explain how you determined your answer.
   d. What does the point of intersection represent in this problem situation?
   e. Verify your answer to part (c) by solving the system algebraically.
   f. Does the solution make sense in terms of this problem situation? Explain your reasoning.
   g. Is this system of equations consistent or inconsistent? Explain your reasoning.
3. Identify whether each system is consistent or inconsistent. Explain your reasoning.
   a. $\begin{cases} -3x + 4y = 3 \\ -12x + 16y = 8 \end{cases}$
   b. $\begin{cases} 7x + 3y = 0 \\ 14x + 6y = 0 \end{cases}$
   c. $\begin{cases} 6x + y = 1 \\ -6x - 4y = -4 \end{cases}$

Stretch

Solve the system of equations shown. Explain your reasoning.

$$\begin{cases} 3x + 5y = 18 \\ y = |x - 4| \end{cases}$$

Review

1. Solve and graph each compound inequality.

   a. $10 < x - 10 \leq 25$

   b. $2x - 11 \leq -5$ or $\frac{1}{3}x + 5 \geq 2$

2. Solve the equation and check your solution.

$$\frac{3}{4}x - 11 = 4 + \left(-\frac{3}{4}x + 3\right)$$

3. Consider the equation $\frac{2}{5}x - 2y = 14$. Write the equation in general form and identify the slope and y-intercept.

4. Determine the linear regression equation for each data set. Which regression equation is the better fit? Explain your reasoning.

Set A

x	y
1	12
2	11
5	30
7	39
10	50

Set B

x	y
12	3
10	9
8	11
5	14
0	0