

Warm Up

Rewrite each fraction in lowest terms.

1. $\frac{12}{3}$

2. $\frac{3}{12}$

3. $\frac{0}{3}$

4. $\frac{3}{0}$

Learning Goals

- Graph rational functions.
- Compare rational functions in multiple representations.
- Compare the basic rational function to various basic polynomial functions.
- Analyze the key characteristics of rational functions.

Key Terms

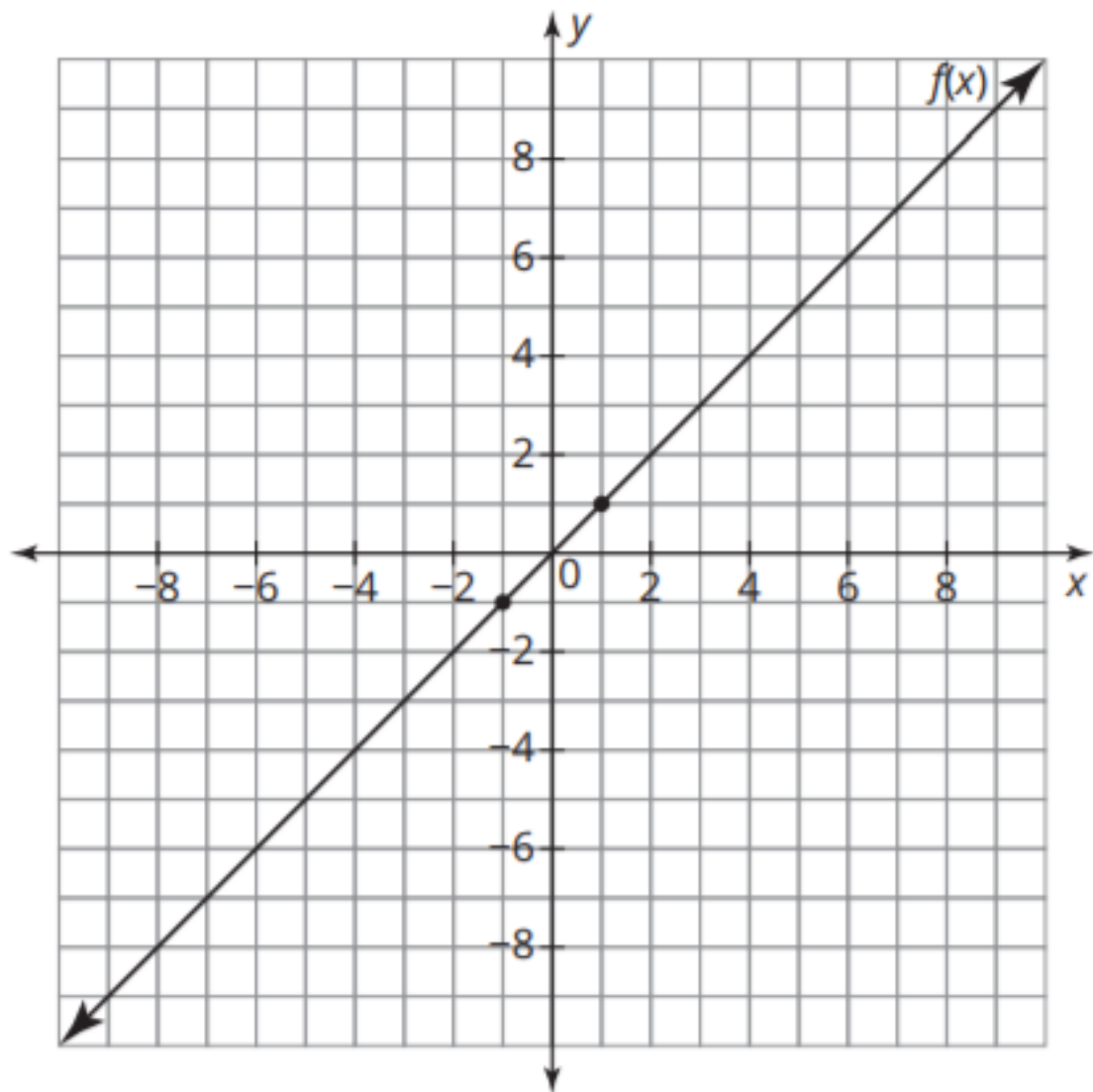
- rational function
- vertical asymptote

A decorative graphic consisting of a large purple cloud-like shape with the word "Remember:" written inside in white. Below the cloud are two smaller purple circles of different sizes, one above the other, connected by a thin vertical line.

Remember:

The reciprocal of any number x is $\frac{1}{x}$. For example, the reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of 0.5 is $\frac{1}{0.5}$, or 2.

- 1. List some properties of rational numbers, along with examples.**
- 2. Determine the reciprocals of the rational numbers you wrote in Question 1. Explain why these numbers are reciprocals.**
- 3. Explain what happens when the denominator of a rational number is zero.**



- b. Describe the graph of $g(x)$. How is it similar to the graphs of other functions that you've studied? How is it different?**

The point at $g(0)$ is said to be undefined because it is impossible to divide by 0.

- c. Describe the end behavior of $g(x)$. Explain your reasoning in terms of the graph, equation, and table of values.**

- d. Describe $g(x)$ as x approaches 0 from the left. Explain the output behavior of the function in terms of the equation.**
- e. Describe $g(x)$ as x approaches 0 from the right. Explain the output behavior of the function in terms of the equation.**

2. Henry and Rosie disagree about $g(x) = \frac{1}{x}$.

Henry

The graph and table both clearly show that it is a function.

Rosie

It is not a function. Every input doesn't have an output.

Who is correct? Explain your reasoning.



3. Analyze the key characteristics of $g(x) = \frac{1}{x}$.

- a. Does the graph ever intersect the horizontal line $y = 0$? Explain your reasoning in terms of the graph, table, and equation.

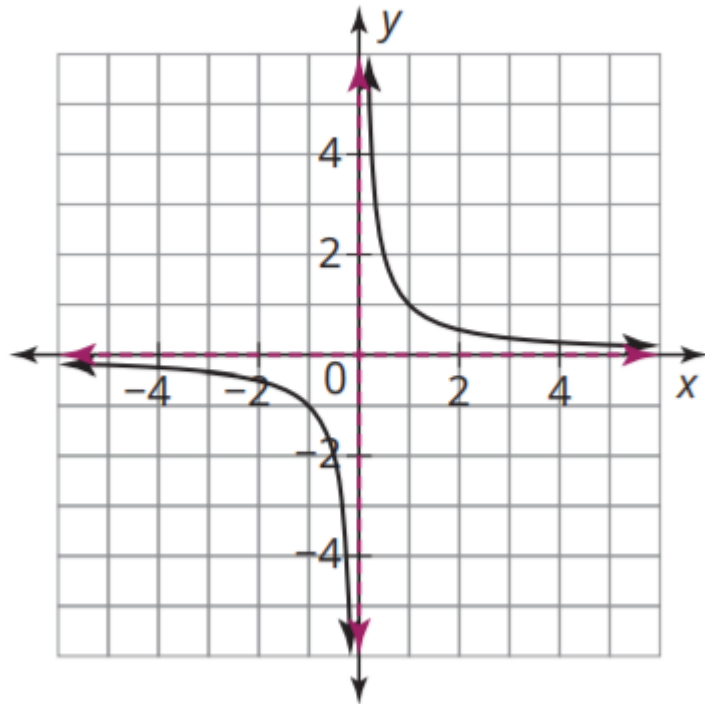
- b. Does the graph ever intersect the vertical line $x = 0$? Explain your reasoning in terms of the graph, table, and equation.

- c. Describe the domain and range of $g(x)$.

The function $g(x) = \frac{1}{x}$ is an example of a *rational function*. A **rational function** is any function that can be written as the ratio of two polynomials. It can be written in the form $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomial functions and $Q(x) \neq 0$. You have already seen some specific types of rational functions. Linear, quadratic, cubic, and higher-order polynomial functions are types of rational functions.

All polynomials are rational functions.
Remember $Q(x)$ can be equal to 1.

The function $g(x) = \frac{1}{x}$ has a *vertical asymptote* at $x = 0$. A **vertical asymptote** is a vertical line that a function gets closer and closer to, but never intersects. The asymptote does not represent points on the graph of the function. It represents the output value that the graph approaches. An asymptote occurs for input values that result in a denominator of 0.



A graph cannot cross its vertical asymptote because the function is undefined at that value.