1. **Sarah determines the vertical asymptotes for the function $f(x) = \frac{1}{2x^2 - 14x - 16}$.**

Sarah

The terms in the denominator have a common factor of 2, so I factored it out first. Then I factored the remaining quadratic.

$$f(x) = \frac{1}{2(x^2 - 7x - 8)} = \frac{1}{2(x - 8)(x + 1)}$$

Vertical asymptotes occur when the denominator is zero. So, the asymptotes will occur when $x - 8 = 0$ and when $x + 1 = 0$. Therefore, the asymptotes occur at $x = 8$ and $x = -1$.

Is Sarah correct? Explain your reasoning.

2. Analyze each rational function. Use algebra to determine the vertical asymptote(s).

a. $f(x) = \frac{5}{7x - 35}$

b. $g(x) = \frac{1}{x(x - 2)(2x + 3)}$

c. $h(x) = \frac{10}{x^2 - 3x - 10}$

d. $h(x) = \frac{x}{2x^2 + 9x + 4}$

e. $h(x) = \frac{7}{x^4 - 1}$

f. $f(x) = \frac{2}{x^2 + 2}$

g. $g(x) = \frac{x + 2}{(x + 2)(x - 5)}$

Think About:

Something interesting is going on with the function in part (g). We'll explore this concept later in the topic, but for now consider why their asymptotic behavior might be different.

Whatever Floats Your Asymptote

1. Abby and Natasha disagree about functions of the form $p(x) = \frac{a}{x}$ where a is a constant.

Abby

The horizontal asymptote will vary depending on the a-value.

Natasha

All rational functions of this form will have a horizontal asymptote at $y = 0$.

Who is correct? Explain your reasoning.

2. Determine a rational function with the characteristics given.

a. Vertical asymptotes at $x = 3$, $x = -1$, and $x = 0$

b. Vertical asymptotes at $x = -7$, $x = 12$

c. No vertical asymptotes

d. A vertical asymptote at $x = 5$ and a horizontal asymptote at $y = 0$