Warm Up

Rewrite each expression in lowest terms.

1. $\frac{45x}{25x^2}$

2. $\frac{b^2 + b - 30}{3b^2 + 18b}$

3. $\frac{56v - 72}{32v}$

4. $\frac{a + 7}{a^2 + 6a - 7}$

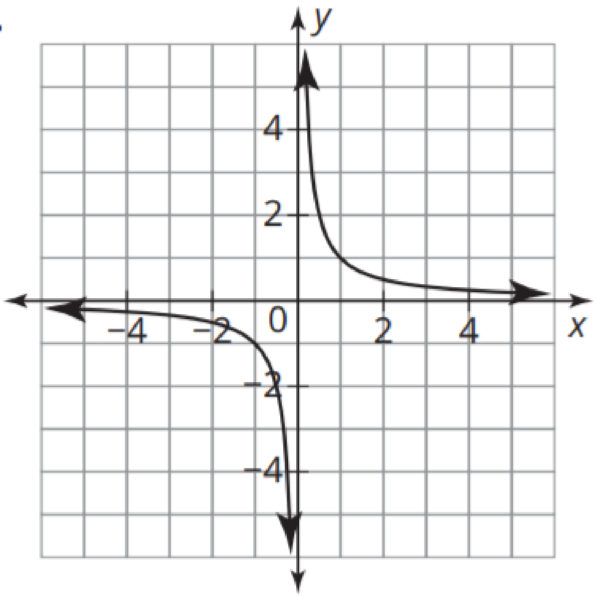
Mend Your Function

1. Without using technology, select a rational equation to represent each graph.

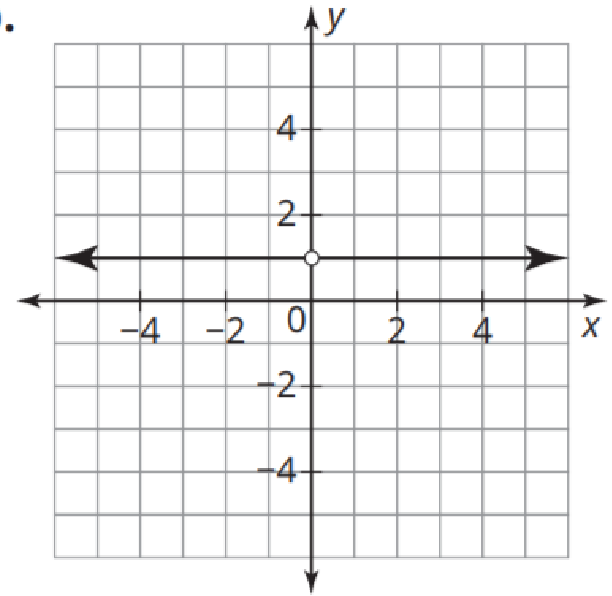
$$y = \frac{1}{x-2} \qquad y = \frac{1}{x} \qquad y = \frac{x^2}{x} \qquad y = \frac{x-2}{x-2}$$

$$y = \frac{x^3}{x} \qquad y = \frac{(x-2)^2}{x-2} \qquad y = \frac{x}{x} \qquad y = \frac{(x-2)^3}{x-2}$$

a.



b.

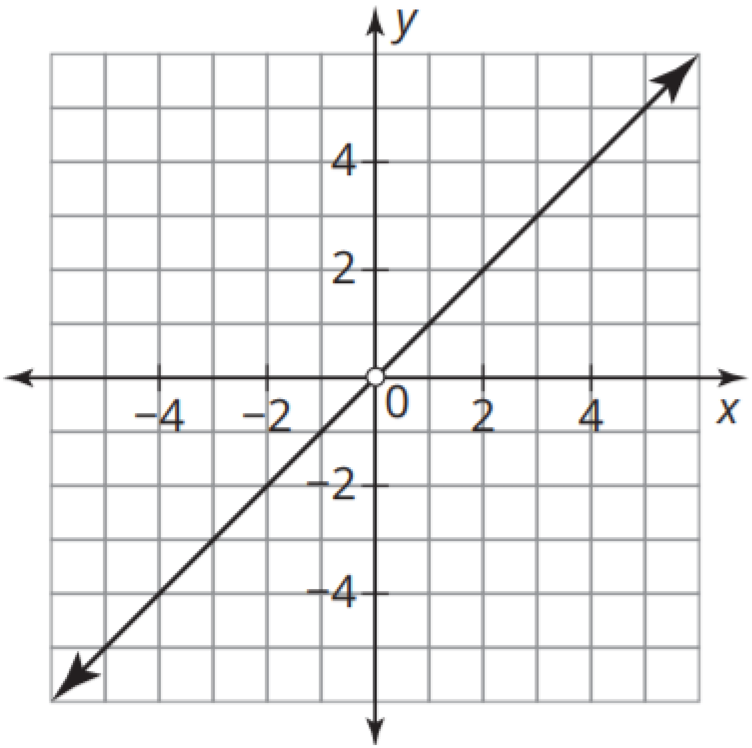


$y = \frac{1}{x-2}$ $\quad y = \frac{1}{x}$ $\quad y = \frac{x^2}{x}$ $\quad y = \frac{x-2}{x-2}$

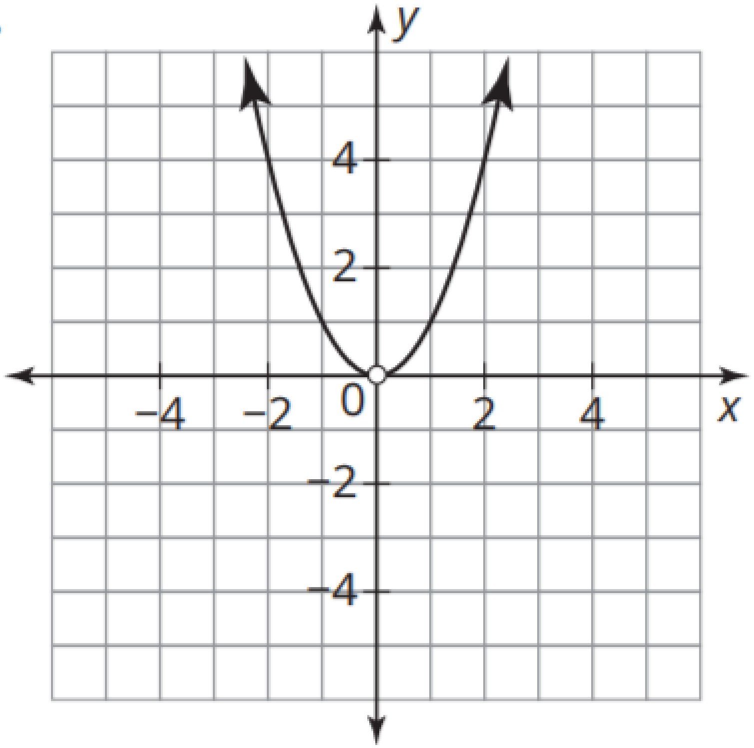
M2-168

$y = \frac{x^3}{x}$ $\quad y = \frac{(x-2)^2}{x-2}$ $\quad y = \frac{x}{x}$ $\quad y = \frac{(x-2)^3}{x-2}$

c.



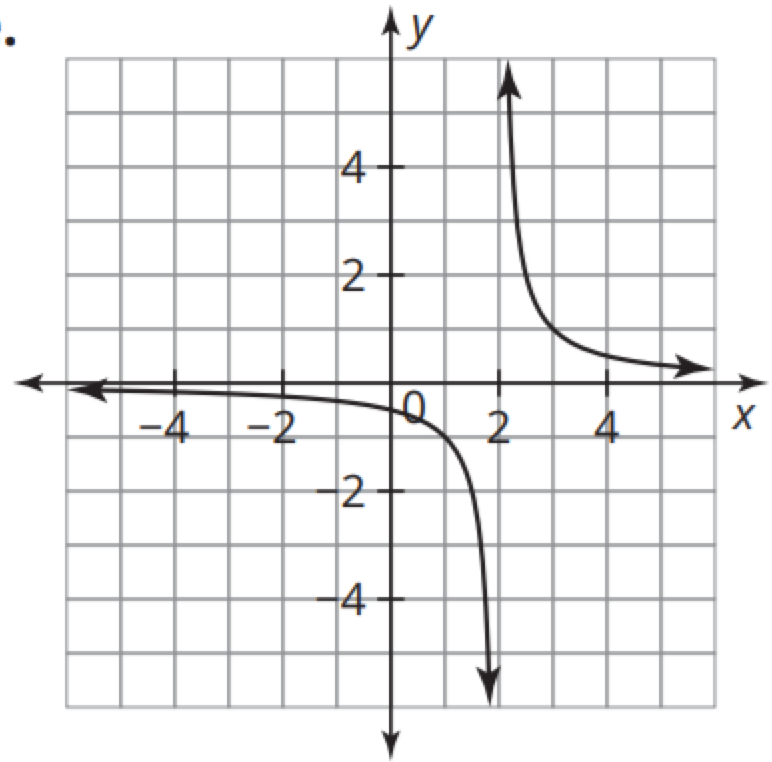
d.



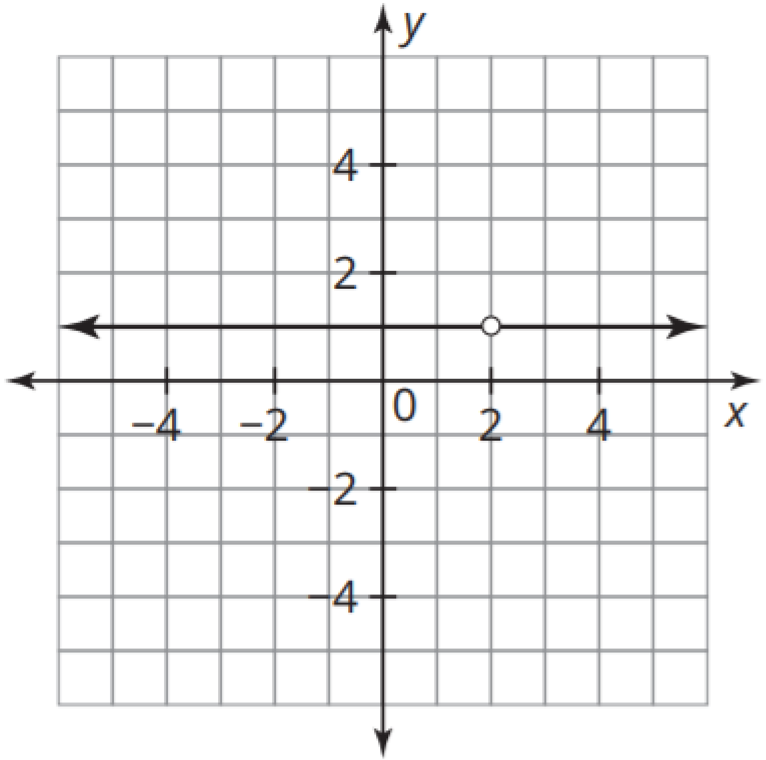
$y = \frac{1}{x - 2}$ $y = \frac{1}{x}$ $y = \frac{x^2}{x}$ $y = \frac{x - 2}{x - 2}$

$y = \frac{x^3}{x}$ $y = \frac{(x - 2)^2}{x - 2}$ $y = \frac{x}{x}$ $y = \frac{(x - 2)^3}{x - 2}$

e.



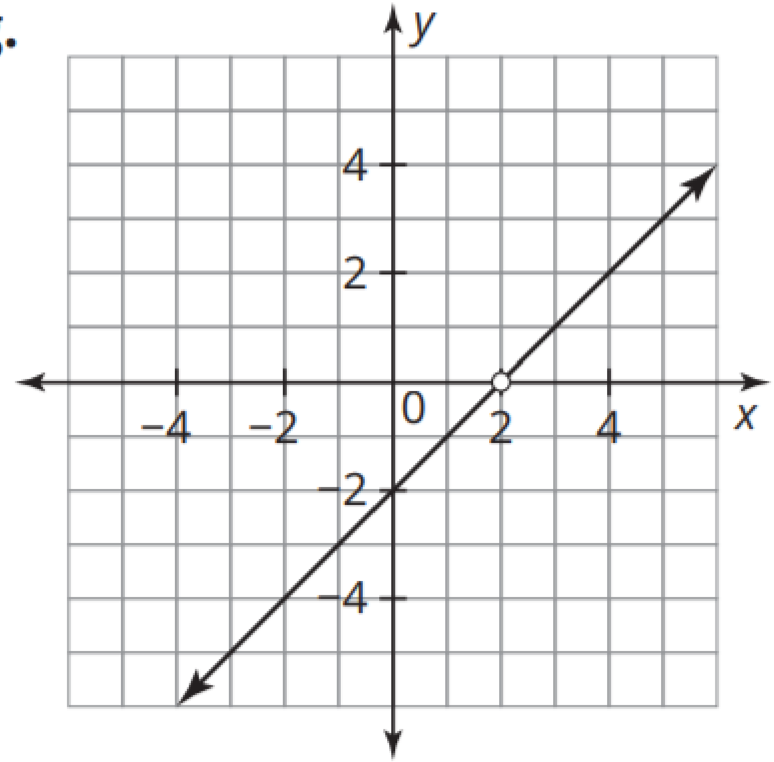
f.



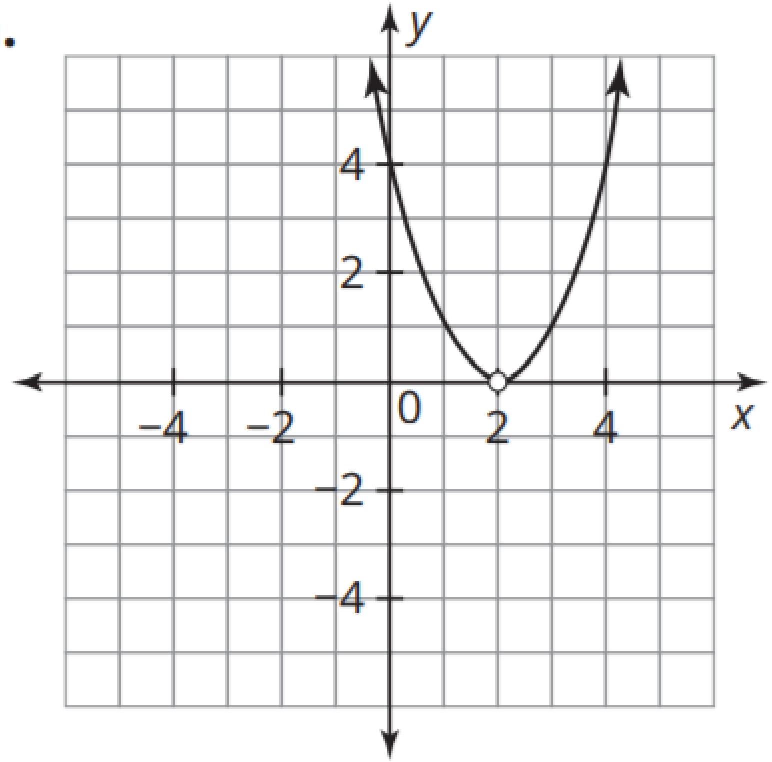
$y = \frac{1}{x-2}$ $\quad y = \frac{1}{x}$ $\quad y = \frac{x^2}{x}$ $\quad y = \frac{x-2}{x-2}$

$y = \frac{x^3}{x}$ $\quad y = \frac{(x-2)^2}{x-2}$ $\quad y = \frac{x}{x}$ $\quad y = \frac{(x-2)^3}{x-2}$

g.



h.



2. Which functions have asymptotes and which functions have "holes" in their graphs? Describe how the structure of the equation determines whether the function will have an asymptote or a "hole". M2-169

$$y = \frac{1}{x-2} \qquad y = \frac{1}{x} \qquad y = \frac{x^2}{x} \qquad y = \frac{x-2}{x-2}$$

$$y = \frac{x^3}{x} \qquad y = \frac{(x-2)^2}{x-2} \qquad y = \frac{x}{x} \qquad y = \frac{(x-2)^3}{x-2}$$

Let's further analyze the graphs of the functions from the Getting Started. Each of these functions is a *discontinuous function*. A **discontinuous function** is a function that does not have a continuous curve—it has points that are isolated from each other.

1. **Compare the graphs of $y = \frac{1}{x-2}$ and $y = \frac{x-2}{x-2}$. How are they the same? How are they different? Describe how the structure of the equation reveals these differences.**

2. **Compare the graphs of $y = \frac{x}{x}$ and $y = \frac{x-2}{x-2}$. How are they the same? How are they different? Describe the similarities and differences in the domain and range in terms of the structure of their equations.**

3. **Without using technology, describe the similarities and differences between the graphs of $y = \frac{x^3}{x^2}$ and $y = x$. Explain your reasoning in terms of the structure of the equations.**

Multiplying the outputs for each input reveals that $(x)\left(\frac{1}{x}\right) = 1$. This graph is a horizontal line that is undefined at $x = 0$. It is undefined at $x = 0$ because this is the value for which an asymptote exists for the factor $\frac{1}{x}$. Similar reasoning can be used to show that for any function $f(x)$, $f(x) \cdot \frac{1}{f(x)} = 1$, with breaks in the graph for all undefined values where $f(x) = 0$. These breaks in the graph are called *removable discontinuities*. A **removable discontinuity** is a single point at which the graph is not defined. Vertical asymptotes and removable discontinuities must be listed as domain restrictions.

This shows graphically why common factors divide to 1. This is why it is not mathematically correct to say that terms "cancel."

Learning Goals

- Identify domain restrictions of continuous and discontinuous rational functions.
- Compare removable discontinuities to vertical asymptotes.
- Rewrite rational expressions.
- Sketch discontinuous rational functions with asymptotes and removable discontinuities.

Key Terms

- discontinuous function
- removable discontinuity