3

Throwing Shade

Graphing Inequalities in Two Variables

Warm Up

Determine if each point is a solution to $y > x$, $y < x$, or $y = x$.

1. (8, −2)
2. (0, 7)
3. (−1, −1)
4. (−4, −3)
5. (9, 9)
6. (−3, −10)

Learning Goals

- Write an inequality in two variables.
- Graph an inequality in two variables on a coordinate plane.
- Determine whether a solid or dashed boundary line is used to graph an inequality on a coordinate plane.
- Interpret the solutions of inequalities mathematically and in the context of real-world problems.

Key Terms

- half-plane
- boundary line

You have graphed linear inequalities in one variable. What does the graph of a linear inequality in two variables look like? How does it compare to the graph of a linear equation?

Making a Statement

Consider each solution statement.

$$x = 2$$

$$x < 2 \qquad x \leq 2$$

$$x > 2 \qquad x \geq 2$$

1. Compare the solution statements. What does each one mean?

2. Choose a solution statement and write a scenario to represent it. Then, modify the scenario so the resulting interpretation is one of the other four solution statements.

ACTIVITY
3.1

Linear Inequalities in Two Variables

1. **Coach Purvis is analyzing the scoring patterns of players on his basketball team. Bena is averaging 20 points per game from scoring on two-point and three-point shots.**

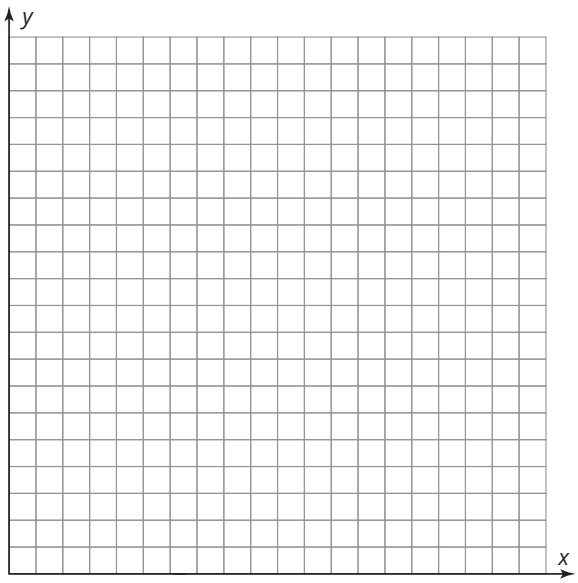
   a. **If she scores 6 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?**

   b. **If she scores 7 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?**

   c. **If she scores 7 two-point shots and 4 three-point shots, will Bena meet her points-per-game average?**

2. **Write an equation to represent the number of two-point shots and the number of three-point shots that total 20 points.**

3. **Graph the equation you wrote on the coordinate plane.**



Ask yourself:

How should you label the graph?

4. **Coach Purvis believes that Danvers High School can win the district playoffs if Bena scores at least 20 points per game.**

   a. **How can you rewrite the equation you wrote in Question 2 to represent that Bena must score at least 20 points per game?**

   b. **Write an inequality in two variables that represents this problem situation.**

Remember:

An inequality is a statement formed by placing an inequality symbol ($<$, $\leq$, $>$, $\geq$) between two expressions.

5. **Complete the table of values. Then, add the ordered pairs in the table to the graph in Question 3. If the number of total points scored does not meet or exceed Bena's points-per-game average, use an "*x*" to plot the point. If the number of total points scored meets or exceeds Bena's points-per-game average, use a dot to plot the point.**

Number of Two-Point Shots Scored	Number of Three-Point Shots Scored	Number of Total Points Scored
4	1	
6	1	
7	1	
8	2	
6	4	
9	5	

6. **What do you notice about your graph?**

7. **What can you interpret about the solutions of the inequality from the graph?**

8. **Choose a different ordered pair located above the line and a different ordered pair that is located below the graph. How do these points confirm your interpretation of the situation? Explain your reasoning.**

9. **Shade the side of the graph that contains the combinations of shots that are greater than or equal to Bena's points-per-game average.**

10. **How do the solutions of the linear equation $2x + 3y = 20$ differ from the solutions of the linear inequality $2x + 3y \geq 20$?**

11. **Does the ordered pair (6.5, 5.5) make sense as a solution in the context of this problem situation? Explain why or why not.**

Like linear equations, linear inequalities take different forms. Each of the linear inequalities in two variables shown represent a different relationship between the variables.

$ax + by < c$ $\qquad$ $ax + by > c$

$ax + by \leq c$ $\qquad$ $ax + by \geq c$

ACTIVITY
3.2
Determining the Graphs of Linear Inequalities

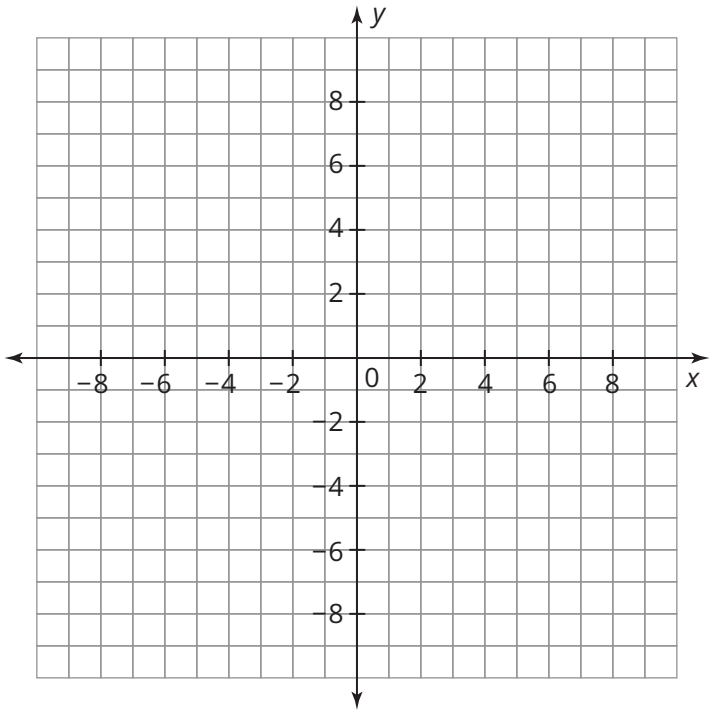
If the inequality symbol is ≤ or ≥, the boundary line is a solid line because all points on the line are part of the solution set. If the symbol is < or >, the boundary line is a dashed line because no point on that line is a solution.

The graph of a linear inequality in two variables is a **half-plane**, or half of a coordinate plane. A **boundary line**, determined by the inequality, divides the plane into two half-planes and the inequality symbol indicates which half-plane contains all the solutions. These solutions are represented by shading the appropriate half-plane.

Consider the linear inequality $y > 4x - 6$. The boundary line that divides the plane is determined by the equation $y = 4x - 6$.

1. Should the boundary line in this graph be a solid line or a dashed line? Explain your reasoning.

2. Graph the inequality on the coordinate plane shown.



After you graph the inequality with either a solid or a dashed boundary line, you need to decide which half-plane to shade. To make your decision, consider the point (0, 0). If (0, 0) is a solution, then the half-plane that contains (0, 0) contains the solutions and should be shaded. If (0, 0) is *not* a solution, then the half-plane that does not contain (0, 0) contains the solutions and should be shaded.

3. Decide which half-plane to shade.

a. Is (0, 0) a solution? Explain your reasoning.

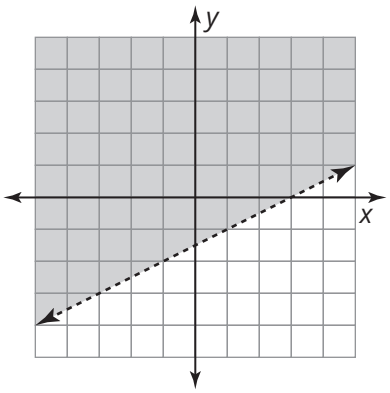
b. Shade the correct half-plane on the coordinate plane.

4. Match each graph to one of the inequalities given. In part (d), graph the inequality that was not graphed in parts (a) through (c).

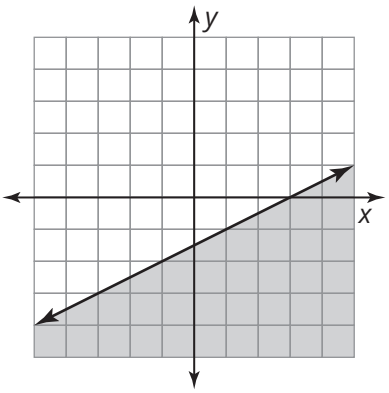
$y \geq \frac{1}{2}x - 3$ $\quad$ $y \leq \frac{1}{2}x - 3$

$y > \frac{1}{2}x - 3$ $\quad$ $y < \frac{1}{2}x - 3$

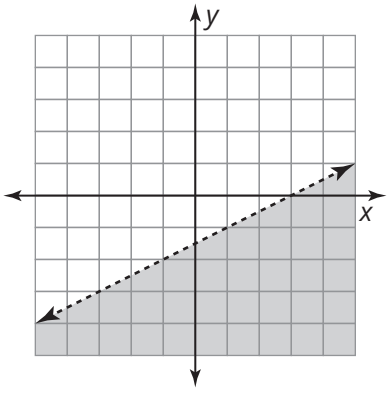
a.



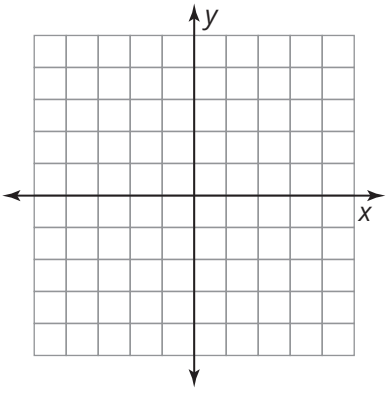
b.



c.



d.



Think about:

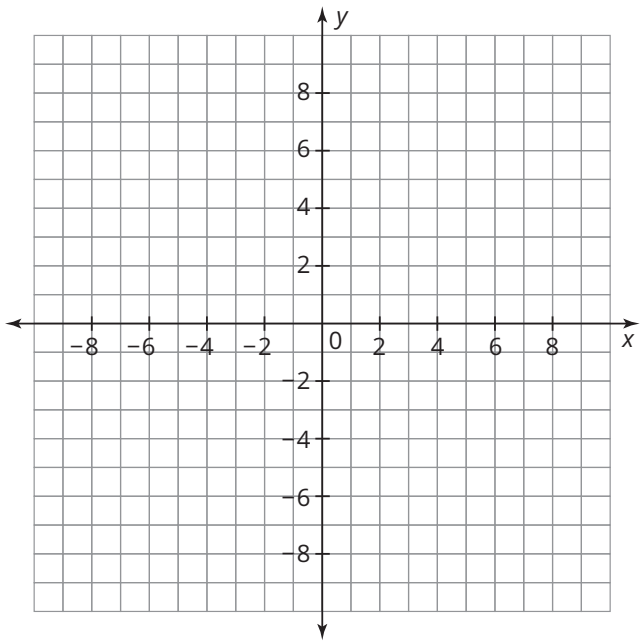
It's a good idea to check points in both half-planes to verify your solution.

Think about:

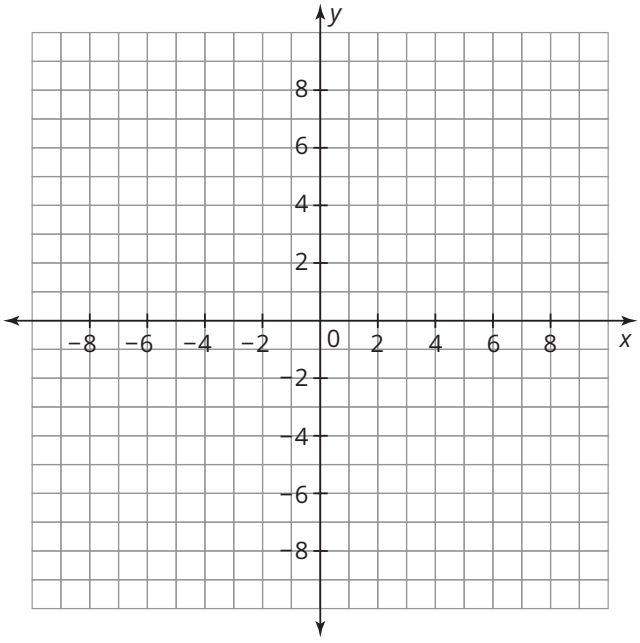
Consider the inequality symbol and which half-plane will be shaded before you test any points.

5. Graph each linear inequality.

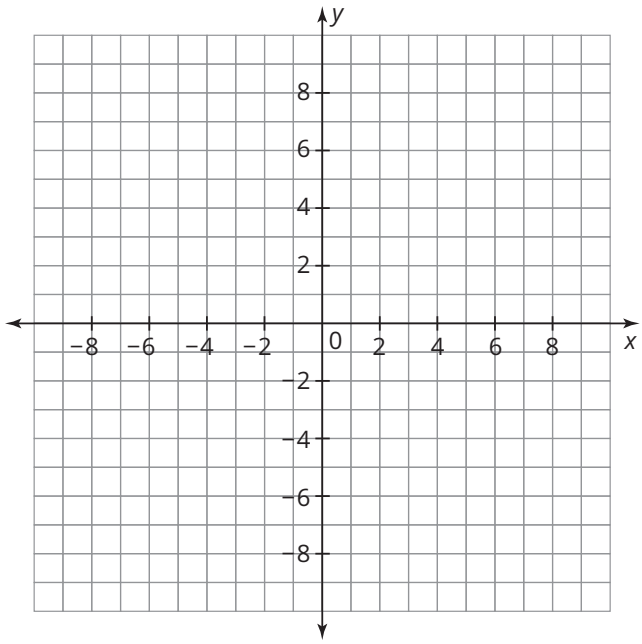
a. $y > x + 3$



b. $y \leq -\frac{1}{3}x + 4$



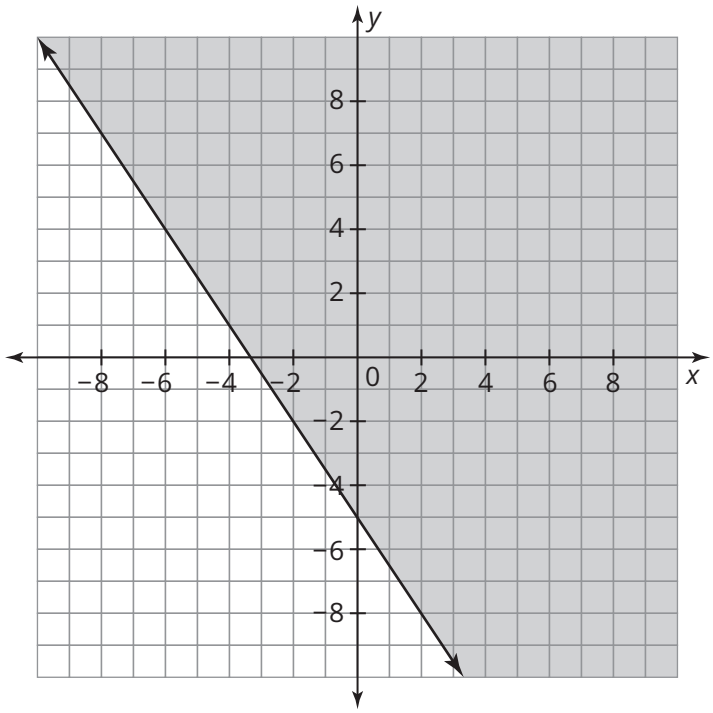
c. $2x - y < 4$



Previously you have written a linear equation given various representations, including two points, one point and the slope, a table of values, or a graph. You can use a similar approach when writing a linear inequality.

Worked Example

Write a linear equality for the graph.



You can use what you have previously learned about the graphs of linear equations to determine that the boundary line is represented by the equation $y = -\frac{3}{2}x - 5$. Now you must decide which inequality symbol should replace the equals sign in the equation.

Since the graph shows a solid boundary line and the half-plane above the line is shaded, use the symbol $\geq$.

$$y \geq -\frac{3}{2}x - 5$$

Test a point in the solution set to check the linear inequality.

Test the point (0, 0):

$$0 \stackrel{?}{\geq} -\frac{3}{2}(0) - 5$$

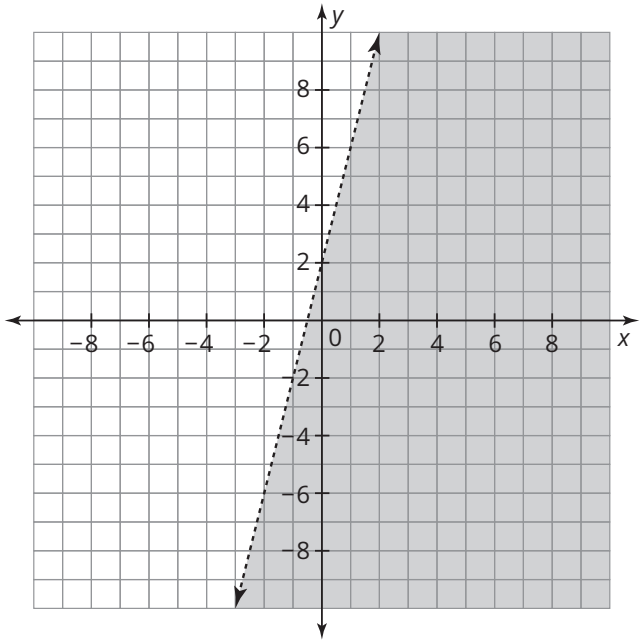
$$0 \geq -5 \checkmark$$

Remember:

The point (0, 0) can be used as a test point unless the boundary line passes through (0, 0).

6. Write a linear inequality for each graph.

a.



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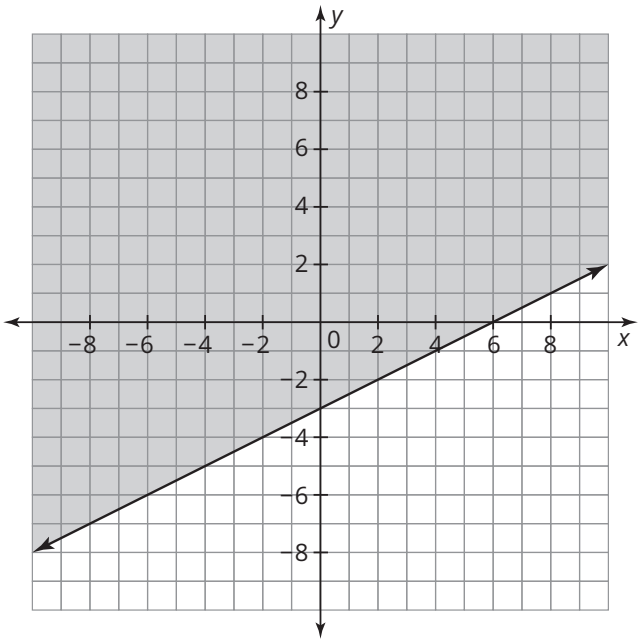

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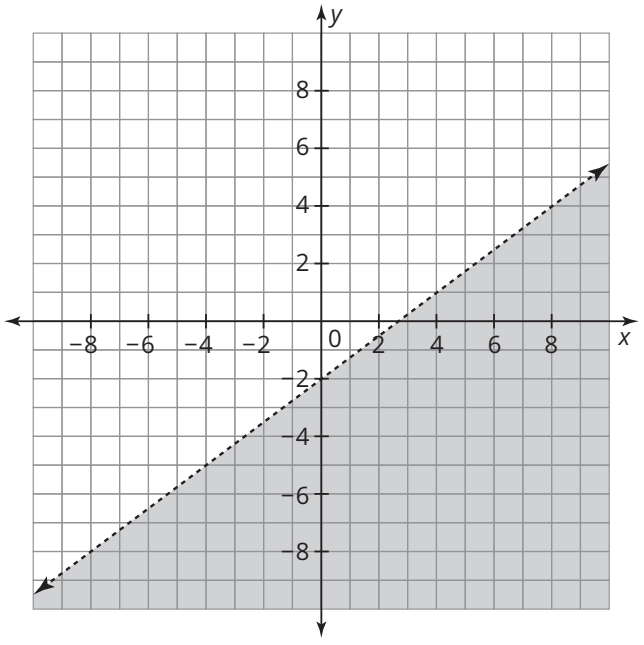


ACTIVITY
3.3

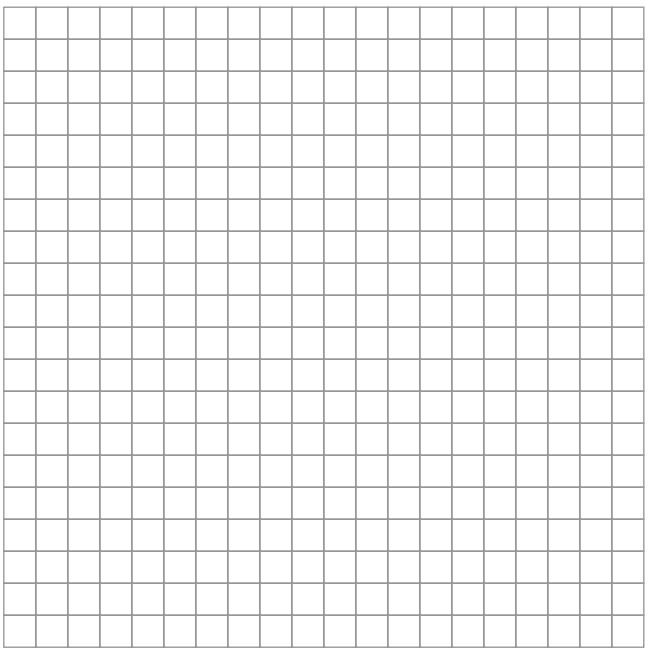
Interpreting the Graph of a Linear Inequality

César has relatives living in both the United States and Mexico. He is given a prepaid phone card worth $50 for his birthday. The table of values shows combinations of minutes for calls within the United States, *x*, and calls to Mexico, *y*, that expend his $50 prepaid phone card.

Length of Calls within United States (minutes)	Length of Calls to Mexico (minutes)
0	100
50	80
140	44
200	20
240	4

1. Write an inequality modeling the number of minutes César can use for calls within the United States and for calls to Mexico.

2. Graph your inequality on the given coordinate grid. Be sure to label your axes.



3. **If César speaks with his aunt in Guadalajara, Mexico, for 70 minutes using his phone card, how long can he speak with his cousin in New York using the same card?**

4. **Can César call his uncle in San Antonio for 100 minutes and also call his grandmother in Juárez, Mexico, for 80 minutes using his phone card? Explain your reasoning.**

5. **Can César call his brother in Mexico City, Mexico, for 55 minutes and also call his sister in Denver, for 90 minutes using his phone card? Explain your reasoning.**

6. **Interpret the meaning of each.**

   a. **points on the line**

   b. **points above the line**

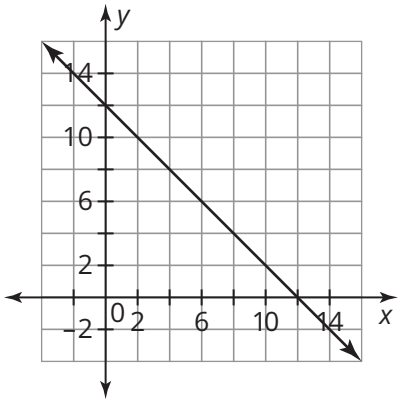
   c. **points below the line**

NOTES

TALK the TALK

There's a Fine Line

Consider the graph of the linear equation $x + y = 12$.



Use the graph to answer each question.

1. **Describe how to graph $x + y < 12$ and choose a point to test this region.**

2. **Describe how to graph $x + y \leq 12$ and choose a point to test this region.**

3. **Describe how to graph $x + y > 12$ and choose a point to test this region.**

NOTES

4. Complete the table.

Equation or Inequality	Description of the Solution Set
$x + y = 0$	
$x + y \geq 0$	
$x + y \leq 0$	
$x + y > 0$	
$x + y < 0$	

Assignment

Write

Describe a half-plane in your own words.

Remember

The graph of a linear inequality in two variables is the half-plane that contains all the solutions. If the inequality symbol is $\geq$ or $\leq$, the graph shows a solid boundary line because the line is part of the solution set. If the symbol is $>$ or $<$, the boundary line is a dashed line because no point on the line is a solution.

Practice

1. Jeremy is working two jobs to save money for his college education. He makes \$8 per hour working for his uncle at Pizza Pie bussing tables and \$10 per hour tutoring peers after school in math. His goal is to make \$160 per week.
   a. If Jeremy works 8 hours at Pizza Pie and tutors 11 hours during the week, does he reach his goal?
   b. Write an expression to represent the total amount of money Jeremy makes in a week from working both jobs. Let x represent the number of hours he works at Pizza Pie and y represent the number of hours he tutors.
   c. After researching the costs of colleges, Jeremy decides he needs to make more than \$160 each week. Write an inequality in two variables to represent the amount of money Jeremy needs to make.
   d. Graph the inequality from part (c).
   e. Is the point (0, 0) in the shaded region of the graph? Explain why or why not.
   f. According to the graph, if Jeremy works 5 hours at Pizza Pie and tutors for 10 hours, will he make more than \$160? Explain why or why not.
   g. Due to days off from school, Jeremy will only be tutoring for 6 hours this week. Use the graph to determine the least amount of full hours he must work at Pizza Pie to still reach his goal. Then show that your result satisfies the inequality.
2. Graph each inequality on a coordinate plane.
   a. $x + 3y > 9$
   b. $2x - 6y \leq 15$
   c. $2x + y < 6$
   d. $3x - y \geq 1$

Stretch

Use what you know about absolute value functions to graph the inequality $y > 2|x - 3| - 5$.

Review

1. Solve each compound inequality. Graph the final solution on a number line.

   a. $-4 \leq 3x + 2 \leq 14$

   b. $\frac{1}{3}x + 3 \geq 4$ or $-x < 2$

2. Solve each system using the Linear Combinations Method.

   a. $\begin{cases} 8x - 6y = -20 \\ -16x + 7y = 30 \end{cases}$

   b. $\begin{cases} x + 3 = -7y + 3 \\ 2x - 8y = 22 \end{cases}$

3. Write the equation of the line that has the given slope and passes through the point given.

   a. $m = \frac{2}{3}$; $(2, -4)$

   b. $m = -4$; $(0.5, 7)$