

2

To the What?

Comparing Exponential Functions

Warm Up

Simplify each expression.

1. $\frac{b^3}{b^0}$

2. $\frac{a \cdot x^5}{a \cdot x^4}$

3. $a \cdot b^2 \cdot a^2 \cdot b^3$

4. $b^{-2} \cdot b^2 \cdot a^0$

Learning Goals

- Write exponential equations to represent situations, tables, and graphs.
- Solve simple exponential equations using common bases.
- Compare exponential equations in different representations.

You have used geometric sequences to define exponential functions. How can you use what you know to compare exponential functions represented as a situation, equation, table, or graph?

Adders and Multipliers

1. The number of friends Eleanor has on her social media account has tripled each month since February for the past 6 months. Now she has over 45,000 friends!

$$\underline{\hspace{1cm}} \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 45,198$$

- a. How many friends did Eleanor have to start? Show how you determined your answer.
- b. What function can you write to represent the number of social media friends Eleanor has since February, given a number of months, x ?

2. Eleanor's dad added 3 friends each month to his social media account in the same time. Now he has 1128 friends.

$$\underline{\hspace{1cm}} + 3 + 3 + 3 + 3 + 3 + 3 = 1128$$

- a. How many friends did Eleanor's dad have to start? Show how you determined your answer.
- b. What function can you write to represent the number of social media friends Eleanor's dad has since February, given a number of months, x ?

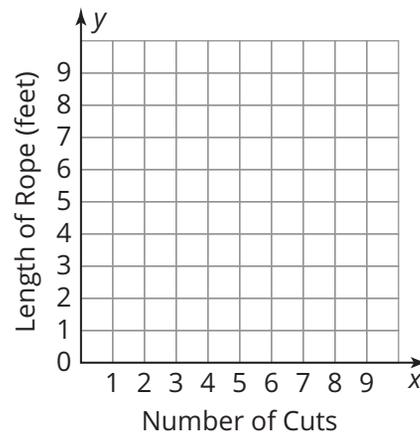
3. Compare the two functions you wrote. What do you notice?



The Amazing Aloysius is practicing one of his tricks. As part of the trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 10-foot rope and then cuts it in half. He takes one of the halves and cuts that piece in half. He keeps cutting the pieces in half until he is left with a piece so small he can't cut it anymore.

1. Complete the table to show the length of rope after each of Aloysius's cuts. Write each length as a whole number, mixed number, or fraction. Then graph the points from the table.

| Number of Cuts | Length of Rope (feet) |
|----------------|-----------------------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |



2. Write the function, $L(c)$, to represent the length of the rope as a function of the cut number, c .
3. Use your function to determine the length of the rope after the 7th cut.

4. Write an exponential function of the form $f(x) = a \cdot b^x$ for each table and graph.

a.

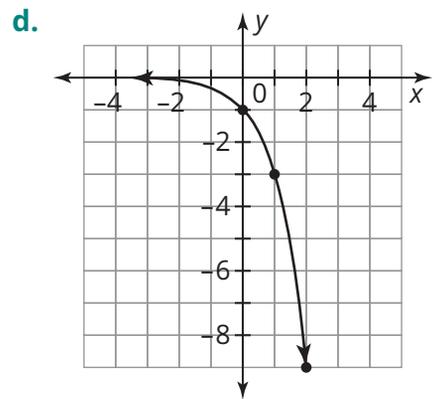
| x | y |
|-----|---------------|
| 0 | 4 |
| 1 | 2 |
| 2 | 1 |
| 3 | $\frac{1}{2}$ |

b.

| x | y |
|-----|----------------|
| -2 | $-\frac{1}{2}$ |
| -1 | -2 |
| 0 | -8 |
| 1 | -32 |

c.

| x | y |
|-----|-----|
| 0 | 1 |
| 1 | 4 |
| 2 | 16 |
| 3 | 64 |



ACTIVITY
2.2

Using Common Bases to Solve Exponential Equations



Suppose a population of rabbits triples every year. The table shows their numbers.

1. Write a function, f , to represent the rabbit population over time.

| Time (years) | Rabbit Population |
|--------------|-------------------|
| 0 | 2 |
| 1 | 6 |
| 2 | 18 |
| 3 | 54 |

2. Use your equation to evaluate the population of rabbits for each number of years.

a. $f(10)$

b. $f(20)$

c. $f(30)$

How long did it take for the population of rabbits to reach a population of 4374? To answer this question, you must solve the equation $4374 = 2(3)^x$. This is equivalent to the equation $2187 = 3^x$.

Worked Example

To solve the exponential equation $2187 = 3^x$, first determine the power of 3 that gives the result of 2187:

$$(3)(3)(3)(3)(3)(3)(3) = 2187$$

$$3^7 = 2187$$

Then rewrite the equation to show common bases:

$$3^7 = 3^x$$

Because the expressions on both sides of the equals sign have the same base, you can set up and solve an equation using the exponents:

$$7 = x$$

So, it will take 7 years for the rabbits to reach a population of 4374.

Remember:

You know that a number is divisible by 3 when the sum of the digits is divisible by 3.

3. Use the method from the worked example to determine approximately how long it will take the rabbit population to reach 1 million. Explain your reasoning.

4. Solve each equation for x .

a. $3^x = 81$

b. $2^{4x} = 1$

c. $4^{8-x} = \frac{1}{64}$

d. $5^{9x} = 1$

e. $\frac{1}{3^{x+5}} = 243$

f. $2^{-x} = \frac{1}{2}$

Comparing Exponential Functions in Different Representations

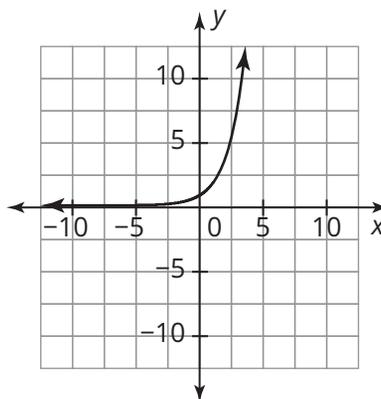


Consider the two exponential functions in each pair. Answer each question and explain your reasoning.

1. Function A

| x | y |
|-----|-----|
| 0 | 1 |
| 1 | 4 |
| 2 | 16 |
| 3 | 64 |

Function B



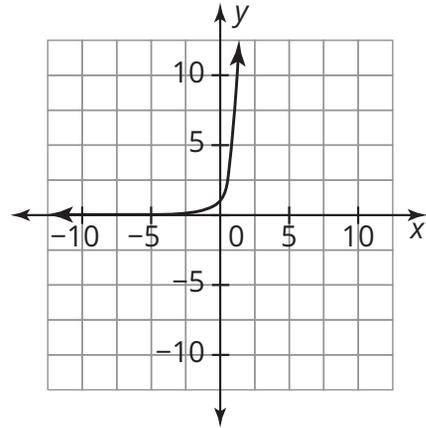
a. Which function has the greater y -intercept?

b. Which function has a greater constant multiplier?

2. Function A

$$f(x) = 2 \cdot 3^x$$

Function B



a. Which function has a greater b -value?

b. Which function has a horizontal asymptote at $y = 0$?

3. Function A

$$f(x) = -2 \cdot 3^x$$

Function B

| x | y |
|-----|-----|
| 0 | 81 |
| 1 | 27 |
| 2 | 9 |
| 3 | 3 |

a. Which function has a greater y -intercept?

b. Which function shows a decrease across its domain?

4. Given each equation, write an exponential equation with the given characteristics. Justify your answers.

a. Write an equation that grows faster than $f(x) = 5^x$.

b. Write an equation with a greater y -intercept than $g(x) = 3 \cdot 2^x$.

c. Write an equation with a horizontal asymptote that is different from the horizontal asymptote of $h(x) = 4 \cdot 3^x$.

TALK the TALK

No Treble

1. Solve each equation for x . Show your work.

a. $9^{2x-5} = 27$

b. $8^{x+6} = 32$

c. $4^{x-5} = 1024^{x-5}$

2. Consider the functions $f(x) = 2 \cdot 2^x$ and $g(x) = 4 \cdot 4^x$. At what point do the graphs of these two functions intersect?

3. Solve for x and explain each step.

$$4^x = \left(\frac{1}{2}\right)^{x-15}$$

Assignment

Write

Explain in your own words how to use common bases to solve an exponential equation.

Remember

You can use what you know about exponential functions to compare them in different representations, such as tables, graphs, equations, and situations.

Practice

1. Complete each table. Write a function that represents the data in the table and explain how you determined your expression.

a.

| x | $f(x)$ | Expression |
|-----|--------|------------|
| 0 | 1 | 3^0 |
| 1 | 3 | |
| 2 | 9 | |
| 3 | | |
| 4 | | |
| 5 | | |
| x | | |

b.

| x | $f(x)$ | Expression |
|-----|--------|------------|
| 0 | 6 | $4^0 + 5$ |
| 1 | 9 | |
| 2 | 21 | |
| 3 | 69 | |
| 4 | 261 | |
| 5 | 1029 | |
| x | | |

2. Solve each equation for the unknown.

a. $4^x = 256$

c. $2^{5-x} = \frac{1}{16}$

e. $4^{x+3} = 4$

g. $-6^{x-2} = \frac{1}{-1296}$

b. $6^{3x} = 216$

d. $3^{-2x} = \frac{1}{729}$

f. $\frac{1}{5^{x+4}} = 625$

h. $\frac{1}{2^{x-6}} = \frac{1}{4}$

Stretch

Describe a way in which you can write a function of the form $f(x) = a \cdot b^x$ as a linear function of the form $f(x) = c$. Explain what the constant multiplier means in this situation.

Review

1. Complete the table.

| Explicit Formula | Exponential Function | Constant Ratio | y-Intercept |
|---|----------------------|----------------|-------------|
| $840 \cdot 3^{x-1}$ | | | |
| $-3 \cdot \left(\frac{1}{5}\right)^{x-1}$ | | | |

2. Solve each equation for x . Provide reasoning to justify each step of your solution method.

a. $10 = -3x + 4 - 2$

b. $1 = 2x - 1 + 2$

3. Rectangle $ABCD$ is shown on the graph.

a. Explain how you can transform the rectangle so that point C is located at the origin.

b. Graph the translated rectangle $A'B'C'D'$ so that point C is located at the origin. Then determine the area of rectangle $A'B'C'D'$.

