Warm Up

Solve each equation for *x*.

1. $y = x^2 - 1$

2. $y = \frac{1}{4}x^2$

3. $y = (x + 4)^3$

The Root of the Matter

You know that the inverse of a power function defined by the set of all points (x, y), or $(x, f(x))$ is the set of all points (y, x), or $(f(x), x)$. Thus, to determine the equation of the inverse of a power function, you can transpose x and y in the equation and solve for y.

Worked Example

Determine the inverse of the power function $f(x) = x^2$, or $y = x^2$.

First, transpose x and y.

$$y = x^2 \longrightarrow x = y^2$$

Then, solve for y.

$$\sqrt{x} = \sqrt{y^2}$$

$$y = \pm\sqrt{x}$$

The inverse of $f(x) = x^2$ is $y = \pm\sqrt{x}$.

1. **Why must the symbol ± be written in front of the radical to write the inverse of the function $f(x) = x^2$?**

2. **Notice that the inverse of the function $f(x) = x^2$ is not written with the notation $f^{-1}(x)$. Explain why not.**

You can use the Horizontal Line Test to recognize that the function $f(x) = x^2$ is not an invertible function. Are there any conditions for which the function $f(x) = x^2$ could pass the Horizontal Line Test?

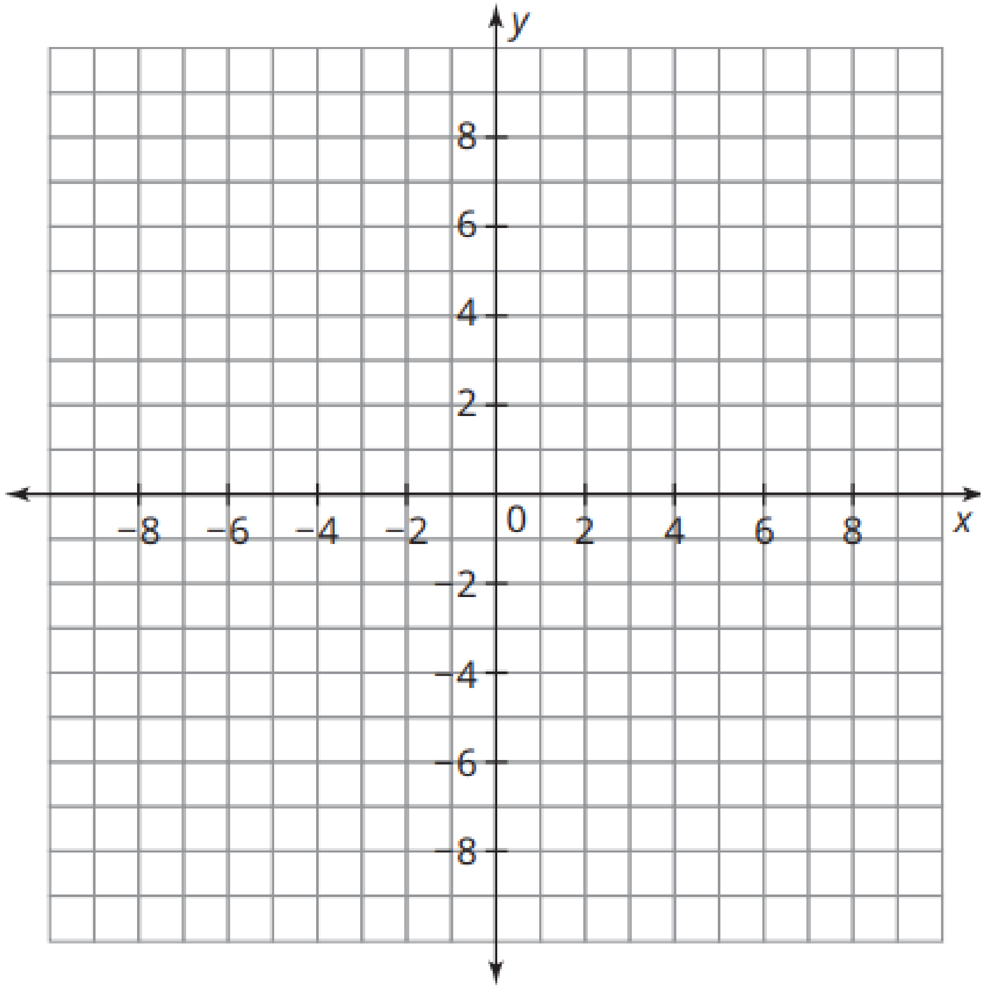
1. The table shows several coordinates of the function $f(x) = x^2$.

a. Use the ordered pairs in the table and what you know about inverses to graph the function and the inverse of the function, $y = \pm\sqrt{x}$. Explain your reasoning.

Ask yourself:

How does each point (x, y) of the function map to the inverse?

x	$f(x) = x^2$
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9



b. What point or points do the two graphs have in common? Why?

2. Describe the key characteristics of each graph:

Function: $f(x) = x^2$

Domain: ____________________

Range: ____________________

***x*-intercept(s):** ____________________

***y*-intercept(s):** ____________________

Inverse of $f(x)$: $y = \pm\sqrt{x}$

Domain: ____________________

Range: ____________________

***x*-intercept(s):** ____________________

***y*-intercept(s):** ____________________

The graph in Question 1 shows that every positive real number has 2 square roots—a positive square root and a negative square root. For example, 9 has 2 square roots, because $3^2 = 9$ and $(-3)^2 = 9$. The two square roots of 9 are 3 and −3.

When you restrict the domain of the power function $f(x) = x^2$ to values greater than or equal to 0, the inverse of the function is called the **square root function** and is written as:

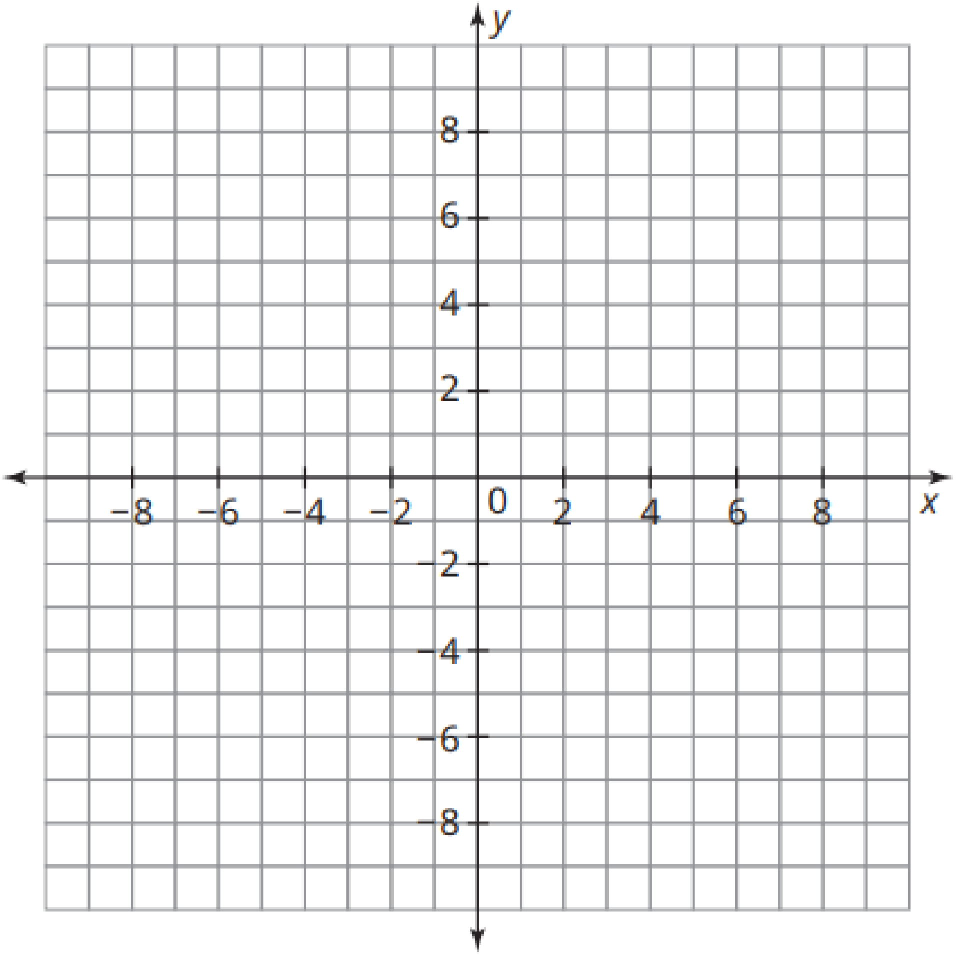
$$f^{-1}(x) = \sqrt{x}, \text{ for } x \geq 0.$$

3. Draw dashed line segments between the plotted points on the function for the restricted domain $x \geq 0$ and the corresponding inverse points.

a. List the ordered pairs of the points you connected.

b. List the ordered pairs of the points that you did not connect. Explain why these points are not connected.

4. Graph the square root function $f^{-1}(x) = \sqrt{x}$ by restricting the domain of $f(x) = x^2$.



Does restricting the domain of the function restrict the range of the inverse?

5. Describe the key characteristics of each function:

Function: $f(x) = x^2$, for $x \geq 0$

Domain: ____________________

Range: ____________________

***x*-intercept(s):** ____________________

***y*-intercept(s):** ____________________

Inverse function: $f^{-1}(x) = \sqrt{x}$

Domain: ____________________

Range: ____________________

***x*-intercept(s):** ____________________

***y*-intercept(s):** ____________________

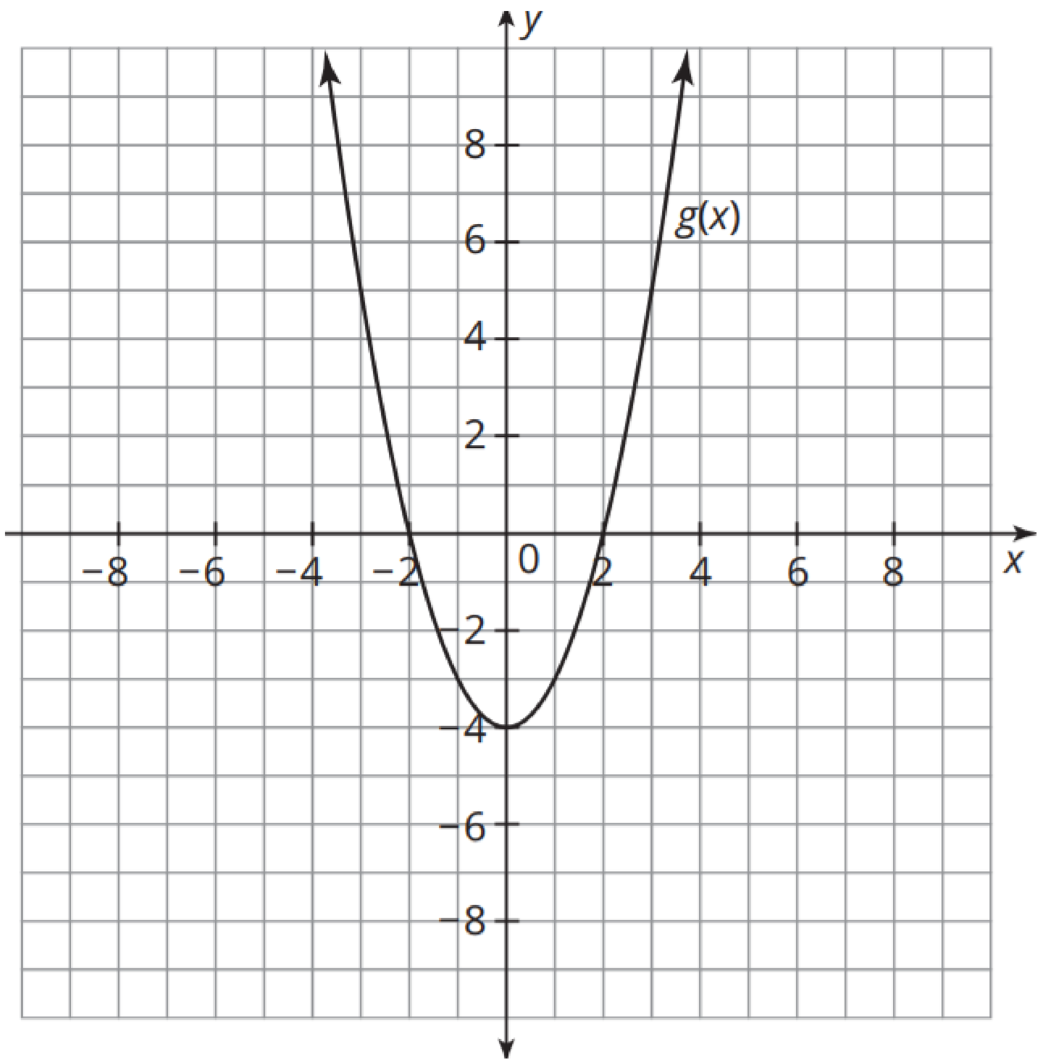
6. **Does the inverse function $f^{-1}(x) = \sqrt{x}$ have an asymptote? Explain your reasoning.**

You've explored the relationship between the function $f(x) = x^2$ and its inverse, both with a domain restriction and without a domain restriction.

7. **Make a conjecture about the relationship between the domain and range of a quadratic function and its inverse.**

Let's look at more quadratic functions to explore domain restrictions and the relationship between the domain and range of a quadratic function and its inverse.

1. Consider the function $g(x) = x^2 - 4$, shown on the coordinate plane.



a. How is $g(x)$ transformed from the basic quadratic function $f(x) = x^2$?

b. Write the equation for the inverse of $g(x)$ and sketch its graph.

c. Is the inverse of $g(x)$ a function? Explain your reasoning.