ACTIVITY 2.4

Inverse by Composition

You know that when the domain is restricted to $x \geq 0$, the function $f(x) = \sqrt{x}$ is the inverse of the power function $g(x) = x^2$. You also know that the function $h(x) = \sqrt[3]{x}$ is the inverse of the power function $q(x) = x^3$.

The process of evaluating one function inside of another function is called the **composition of functions**. For two functions *f* and *g*, the composition of functions uses the output of one function as the input of the other. It is expressed as $f(g(x))$ or $g(f(x))$.

Worked Example

To write a composition of the functions $g(x) = x^2$ and $f(x) = \sqrt{x}$ when the domain of $g(x)$ is restricted to $x \geq 0$, substitute the value of one of the functions for the argument, x, of the other function.

$$f(x) = \sqrt{x} \qquad g(x) = x^2$$

$$f(g(x)) = \sqrt{x^2} = x, \text{ for } x \geq 0$$

You can write the composition of these two functions as $f(g(x)) = x$ for $x \geq 0$.

1. Determine $g(f(x))$ for the functions $g(x) = x^2$ and $f(x) = \sqrt{x}$ for $x \geq 0$.

If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.

2. Are $f(x)$ and $g(x)$ inverse functions? Explain your reasoning.

3. Algebraically determine whether the functions in each pair are inverses. Show your work.

a. $h(x) = \sqrt[3]{x}$ **and** $q(x) = x^3$

b. $k(x) = 2x^2 + 5$ **and** $j(x) = -2x^2 - 5$

4. **Mike said that all linear functions are inverses of themselves because $f(x) = x$ is the inverse of $g(x) = x$.**

Is Mike correct? Explain your reasoning.

The time it takes for one complete swing of a pendulum depends on the length of the pendulum and the acceleration due to gravity.

Acceleration due to gravity is the force acting on an object falling freely under the influence of Earth's gravitational pull.

Acceleration due to gravity is approximately equal to 9.8 $\frac{m}{s^2}$.

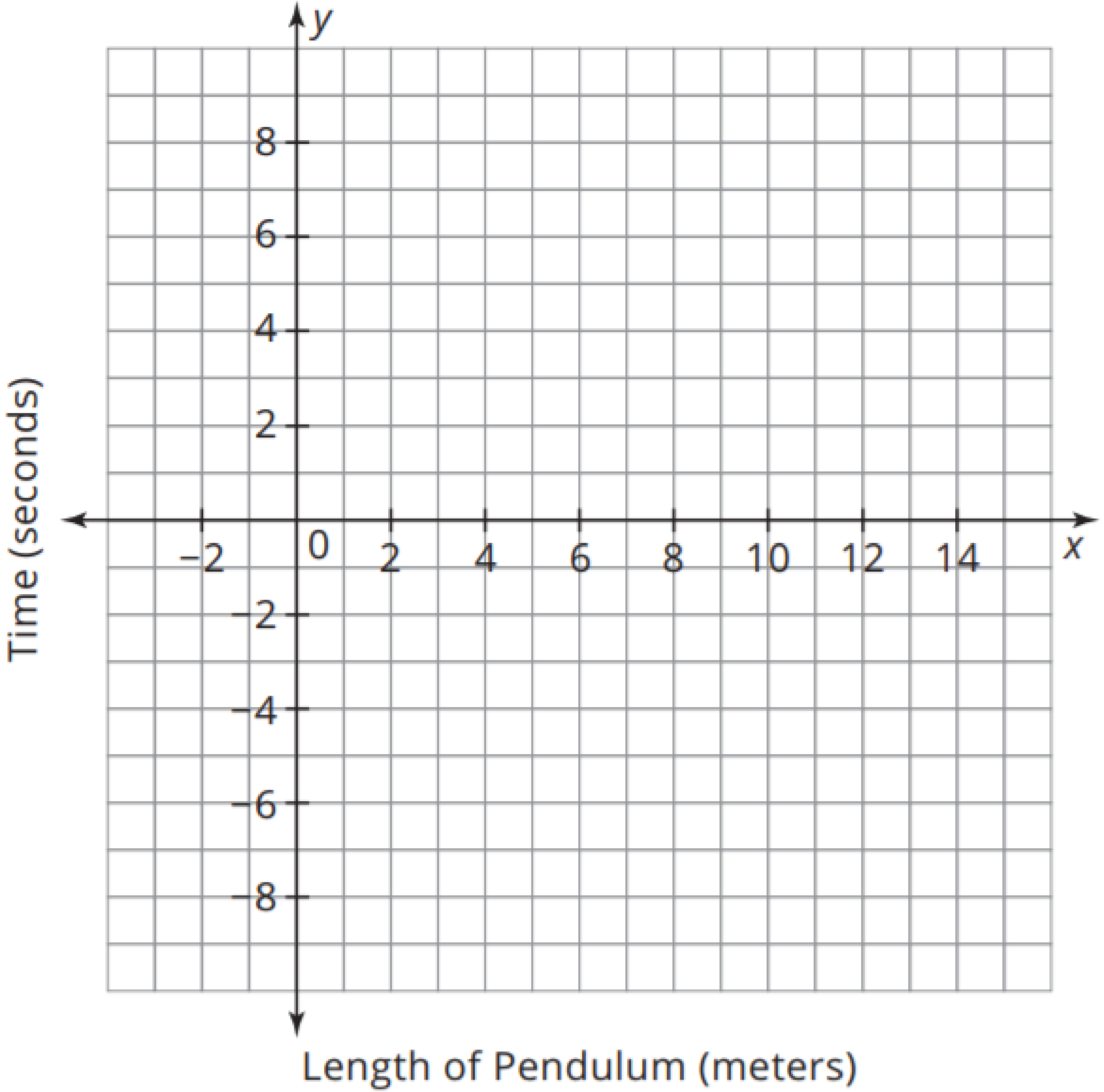
The formula for the time it takes a pendulum to complete one swing is $T = 2\pi\sqrt{\frac{L}{g}}$, where T is time in seconds, L is the length of the pendulum in meters, and g is the acceleration due to gravity in meters per second squared.

1. Write a function $T(L)$ that represents the time of one pendulum swing.

2. Use technology to sketch a graph of the function $T(L)$.

Think about:

Notice that the time for one swing does not depend on the mass of the pendulum.



3. Describe the characteristics of the function, such as its domain, range, and intercepts. Explain your reasoning.

4. How long does it take for one complete swing when the length of the pendulum is 0.5 meter?

ACTIVITY
2.6

Centrifugal Force

The Rotor is a popular amusement park ride shaped like a cylindrical room. Riders stand against the circular wall of the room while the room spins. When The Rotor reaches the necessary speed, the floor drops out and centrifugal force leaves the riders pinned up against the wall.

The minimum speed (measured in meters per second) required to keep a person pinned against the wall during the ride can be determined with the function $s(r) = 4.95\sqrt{r}$, where r is the radius of The Rotor measured in meters.

1. **An amusement park designed a rotor ride with a radius of 2 meters. At what speed does it need to spin?**

2. **The same park decided to build a larger rotor ride with a radius of 4 meters. At what speed does it need to spin?**

3. **Designers at another park have a motor that could spin a rotor ride at 6 meters per second. What is the length of the radius of this ride?**

Complete M3 38

Quiz tomorrow on the topics